Exact Evaluation of Subdivision Surfaces

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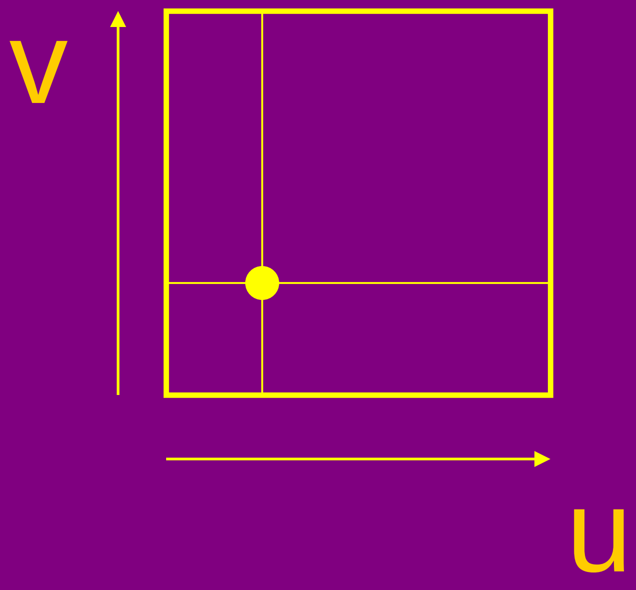
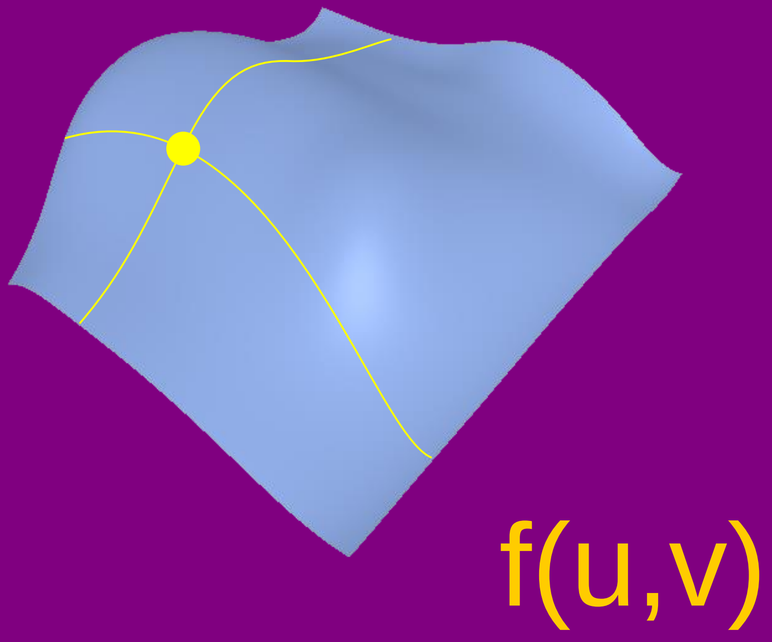
Alias | wavefront

Seattle, WA USA

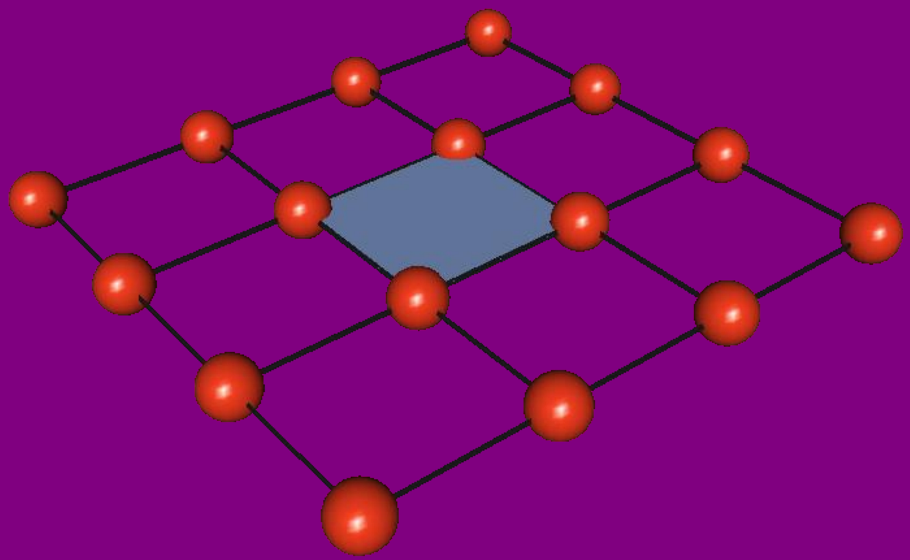
Evaluation of Surfaces



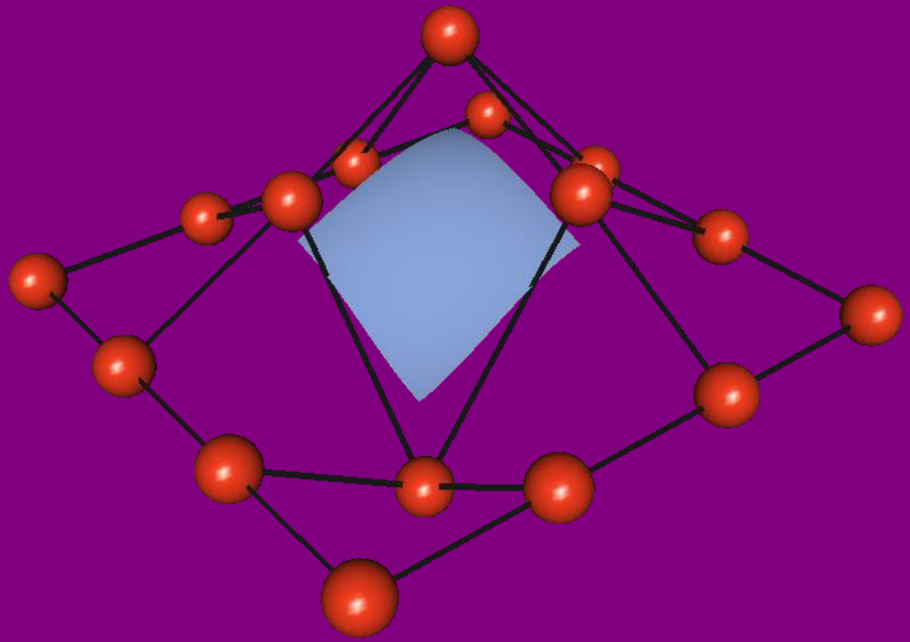


Mapping a 2D square in 3D

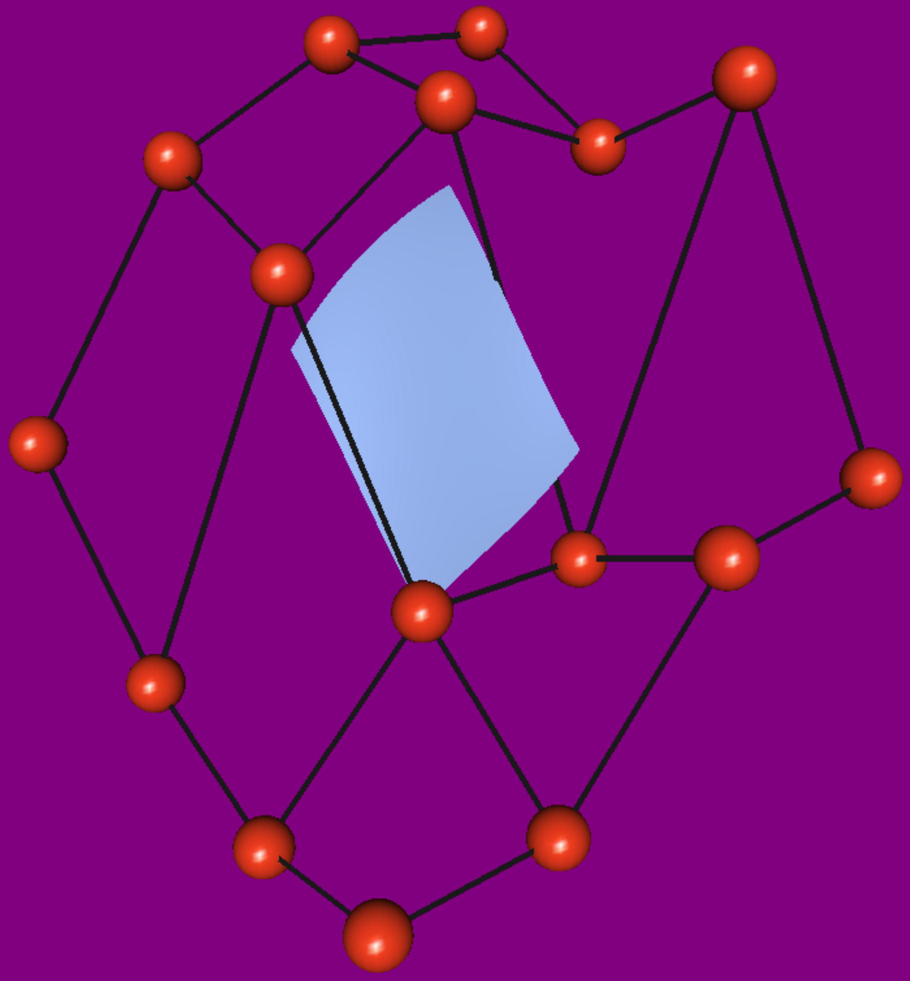
Example: bicubic B-splines



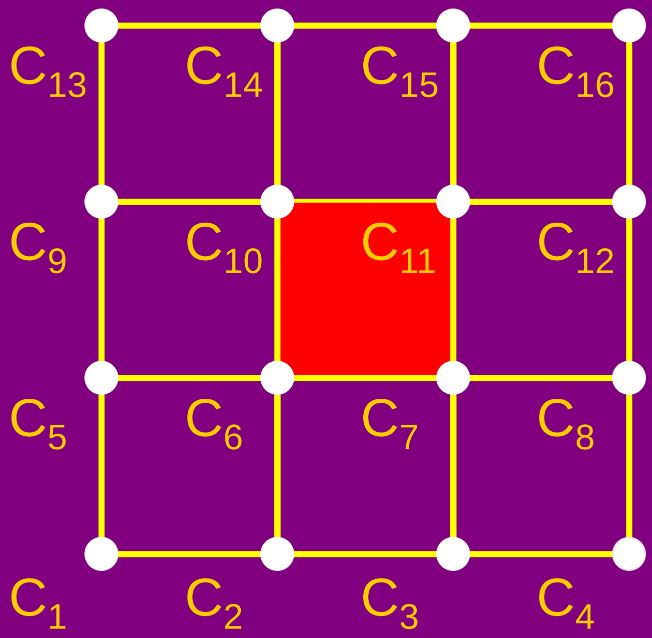
Example bicubic B-splines



Example: bicubic B-splines

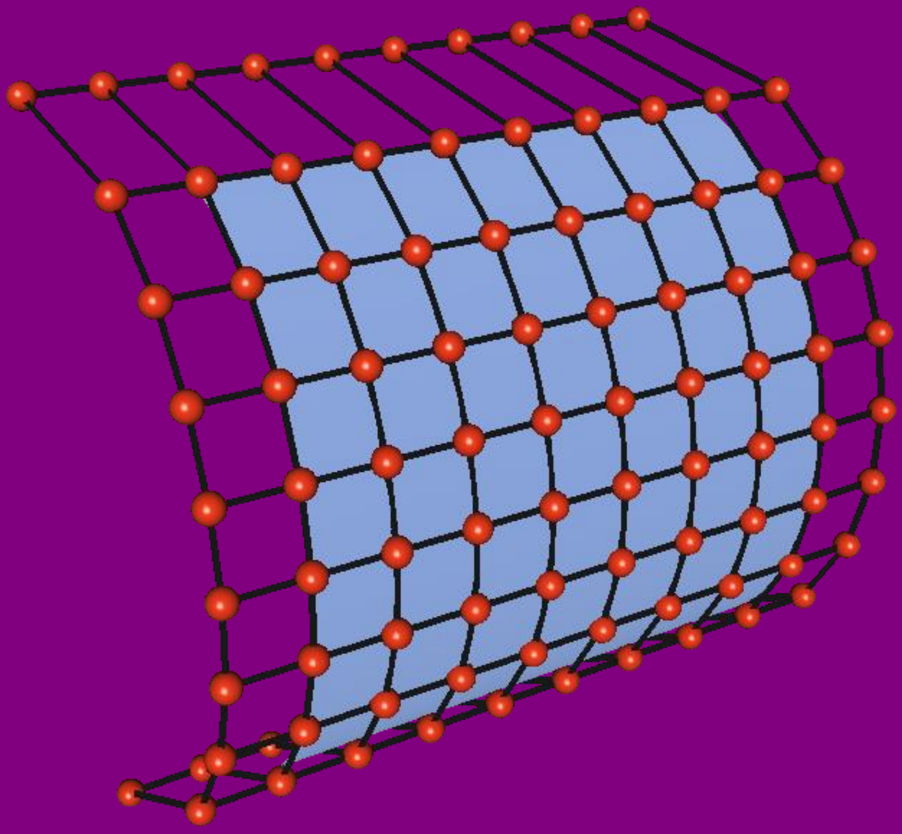


Example: bicubic B-splines

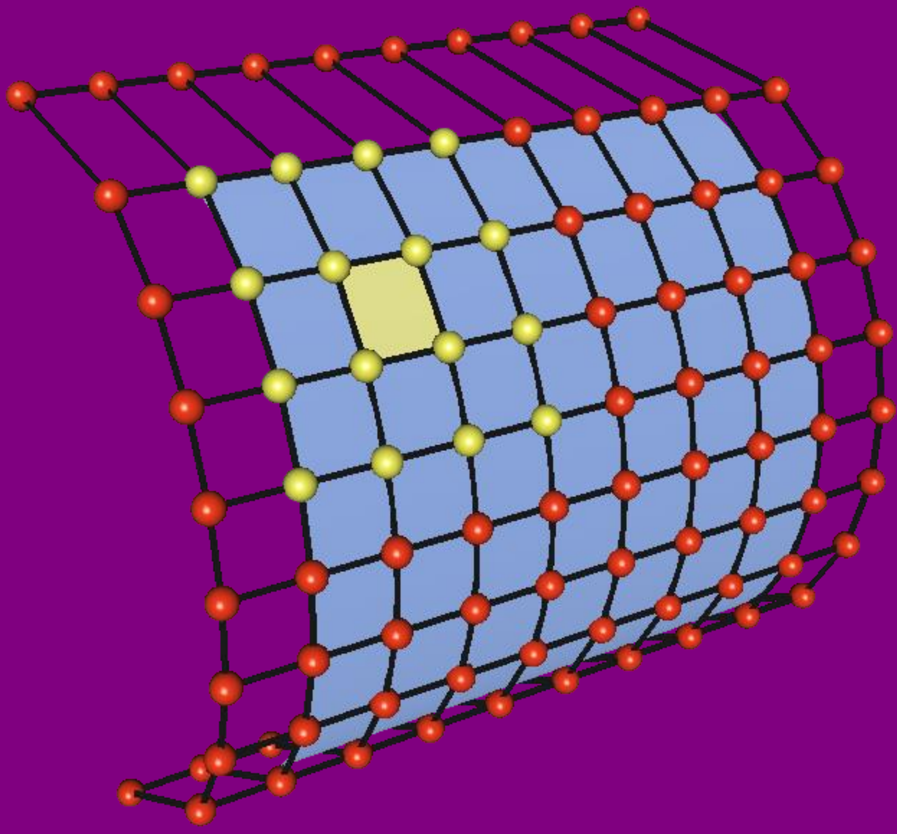


$$f(u,v) = \sum_{i=1}^{16} c_i \, B_i\,(u,v)$$

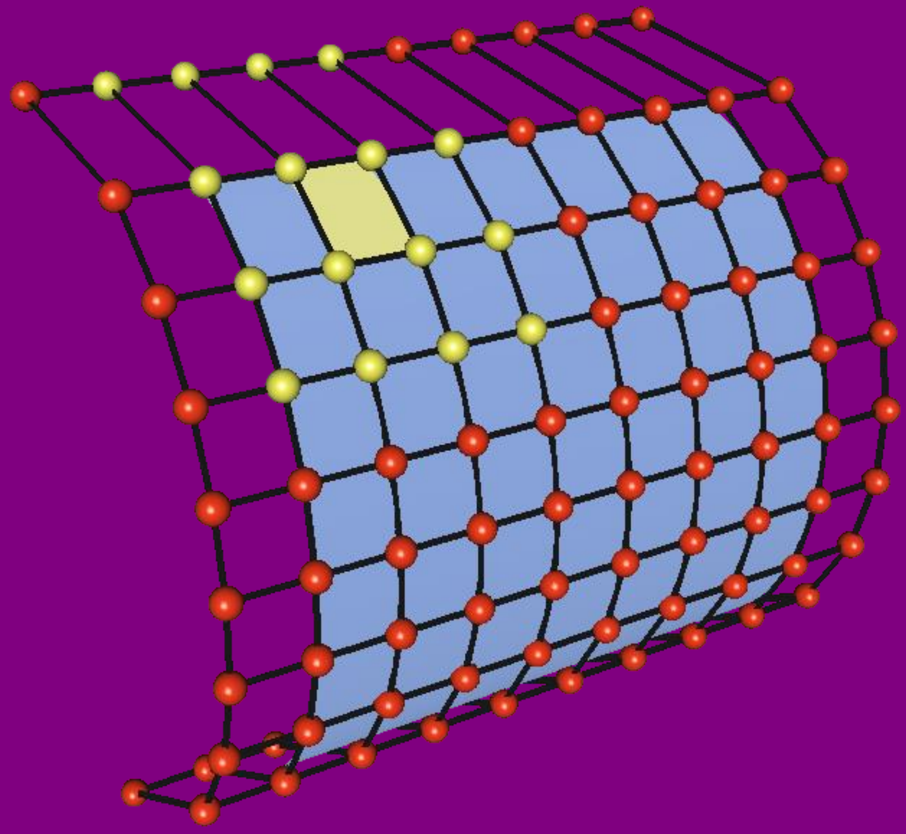
Piece-wise B-splines



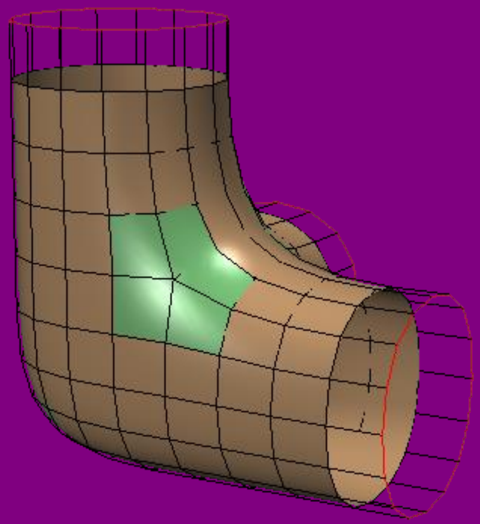
Piece-wise B-splines



Piece-wise B-splines

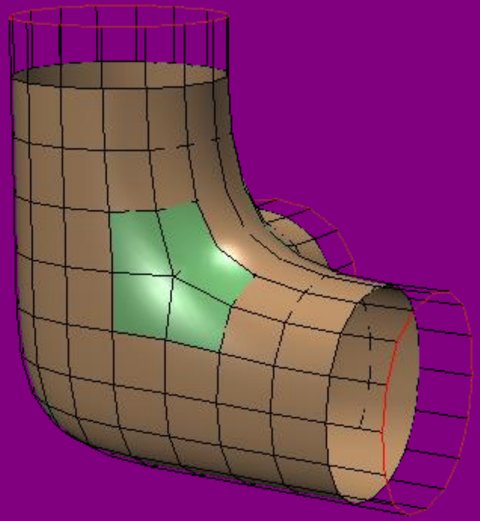


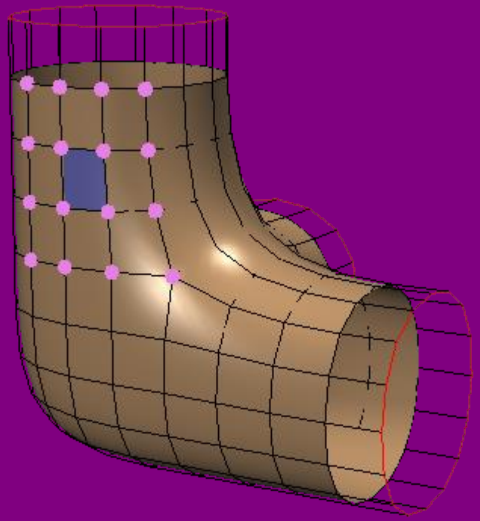
Catmull-Clark Surfaces

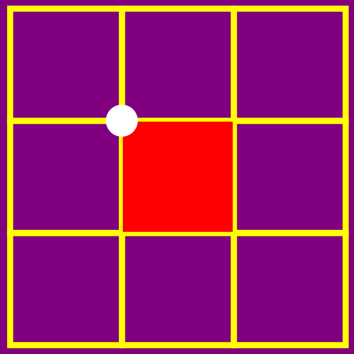


B-splines almost everywhere

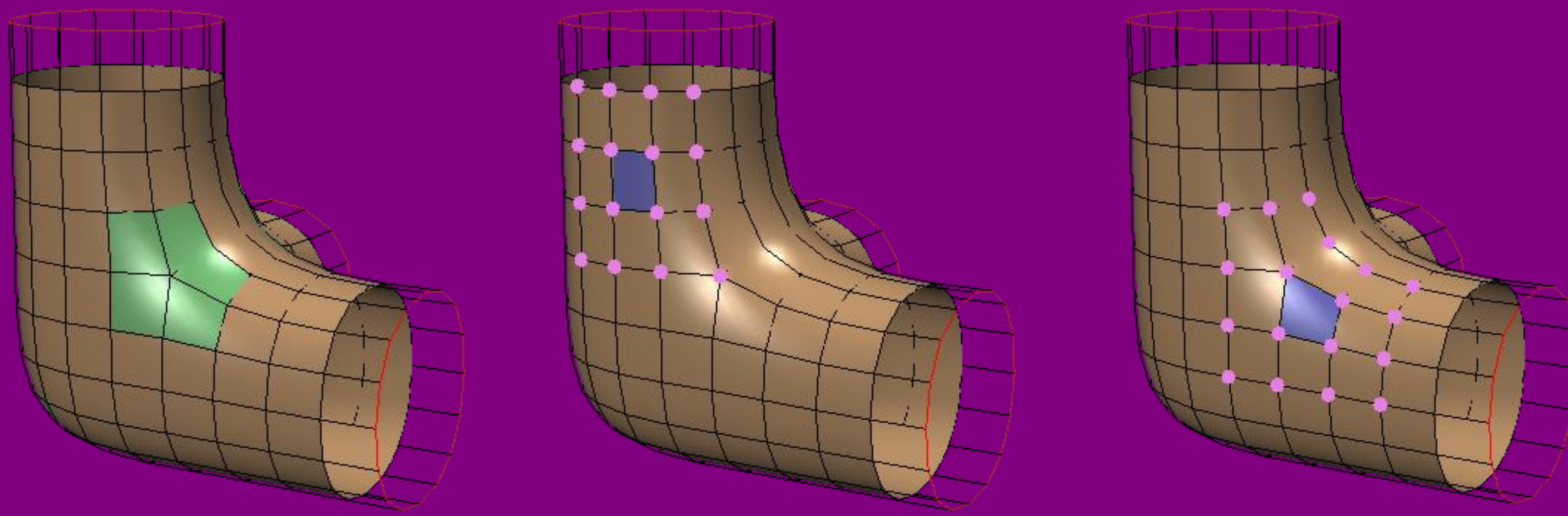
Catmull-Clark Surfaces

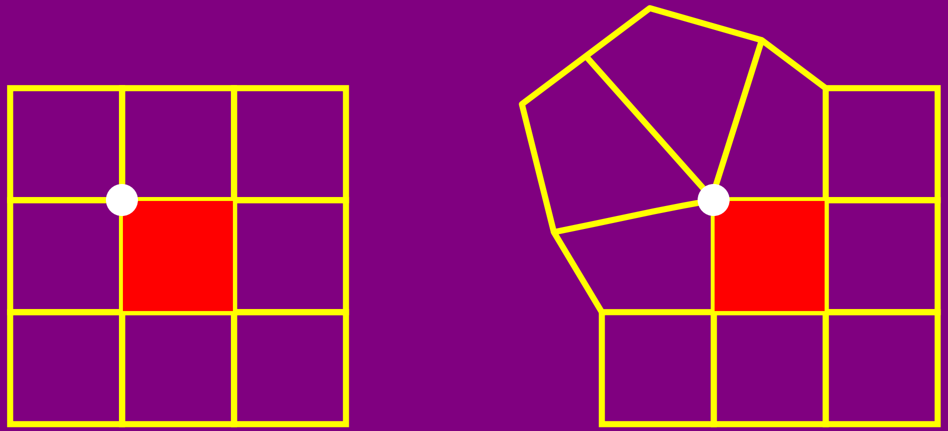






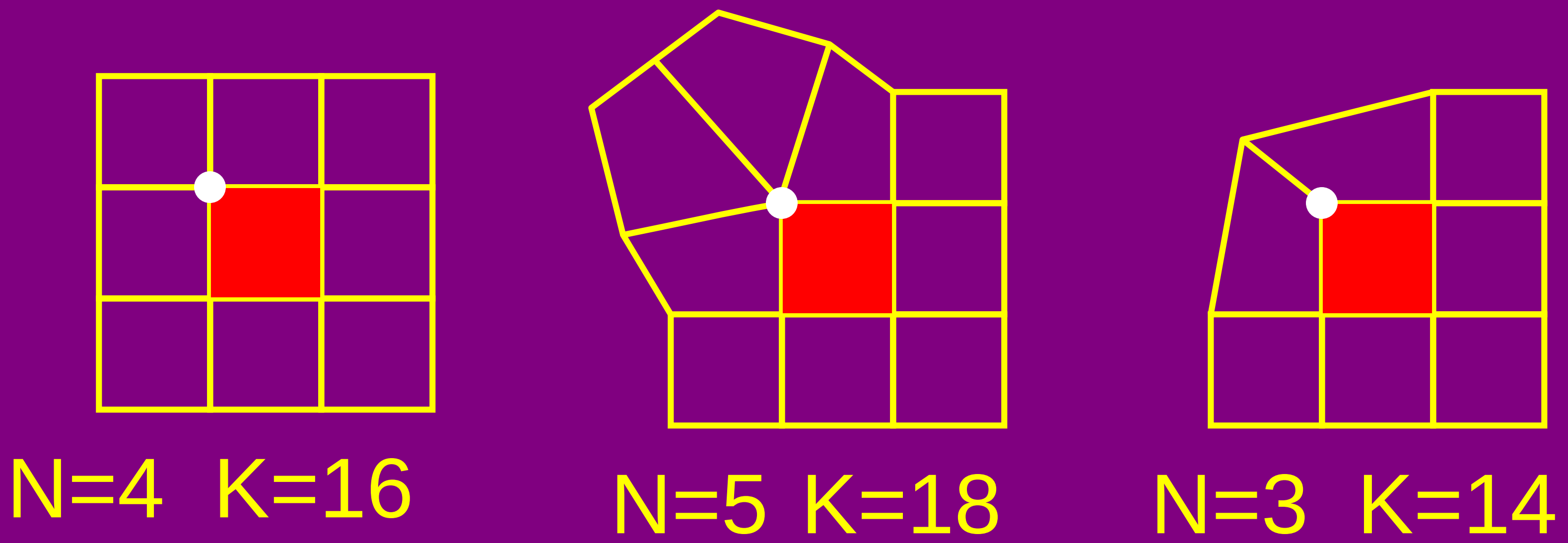
Catmull-Clark Surfaces





Exact Evaluation

Patch defined by K=2N+8 control points



$$f(u,v) = \sum_{i=1}^{K} \alpha_i \, \phi_i (u,v)$$

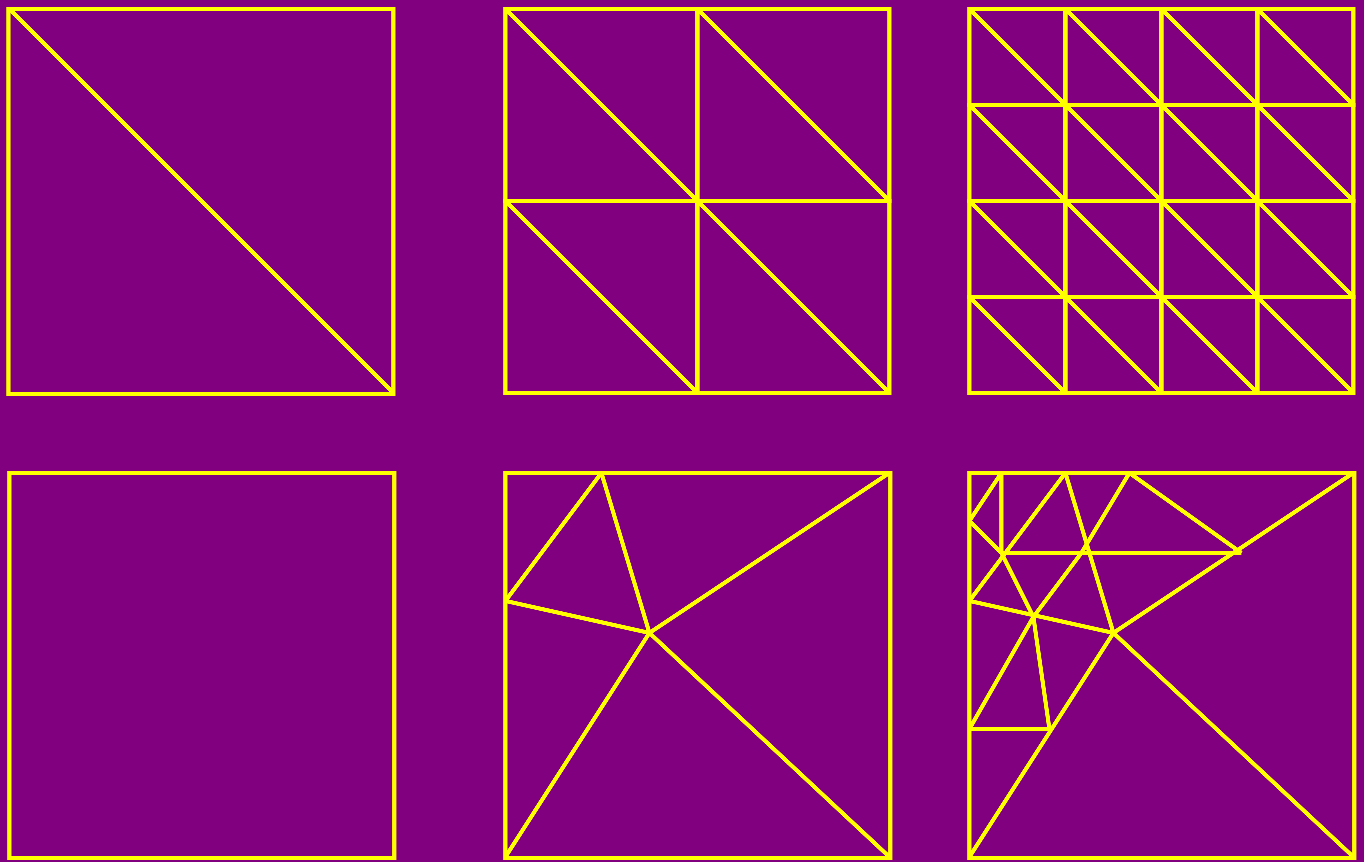
Applications

Reuse algorithms / code:

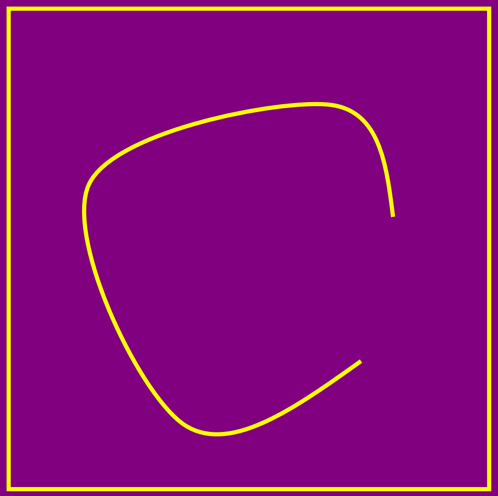
- Rendering
- Curve on Surface
- Intersection
- Integration
- Dynamics (Mandal & Qin)
- …

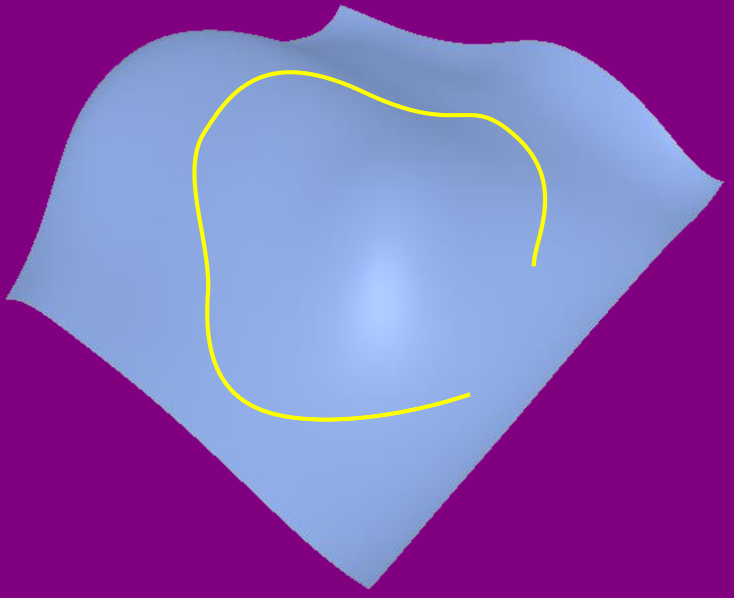
Rendering

Arbitrary Tesselations



Curve on Surface





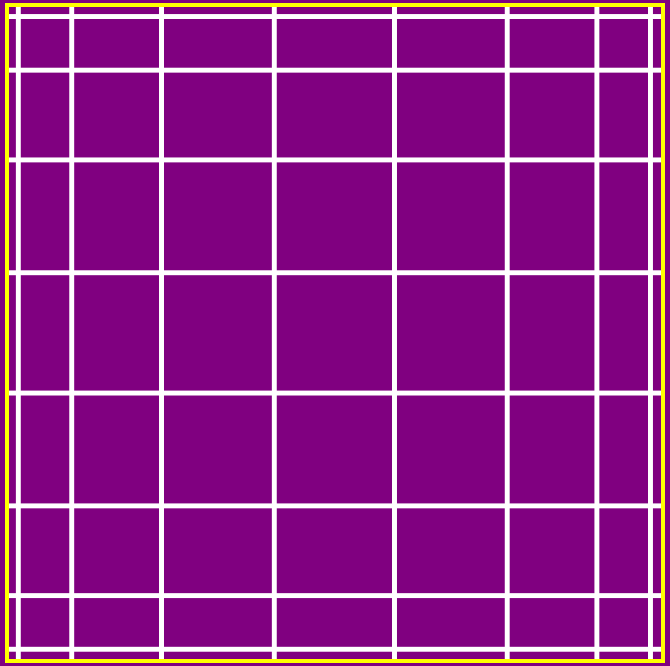
Intersection

Use Newton Method for
Ray - Surface and
Surface - Surface Intersections

Black box code for parametric surfaces

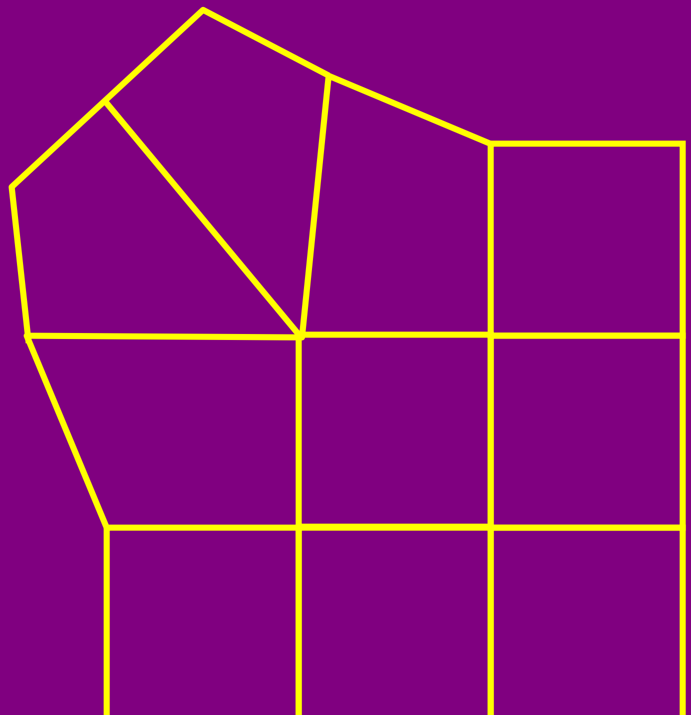
Integration

$$\iint F(u,v)\, du\, dv = \sum_{i,j=1}^{8} w_i\, w_j\, F(u_i, v_j)$$

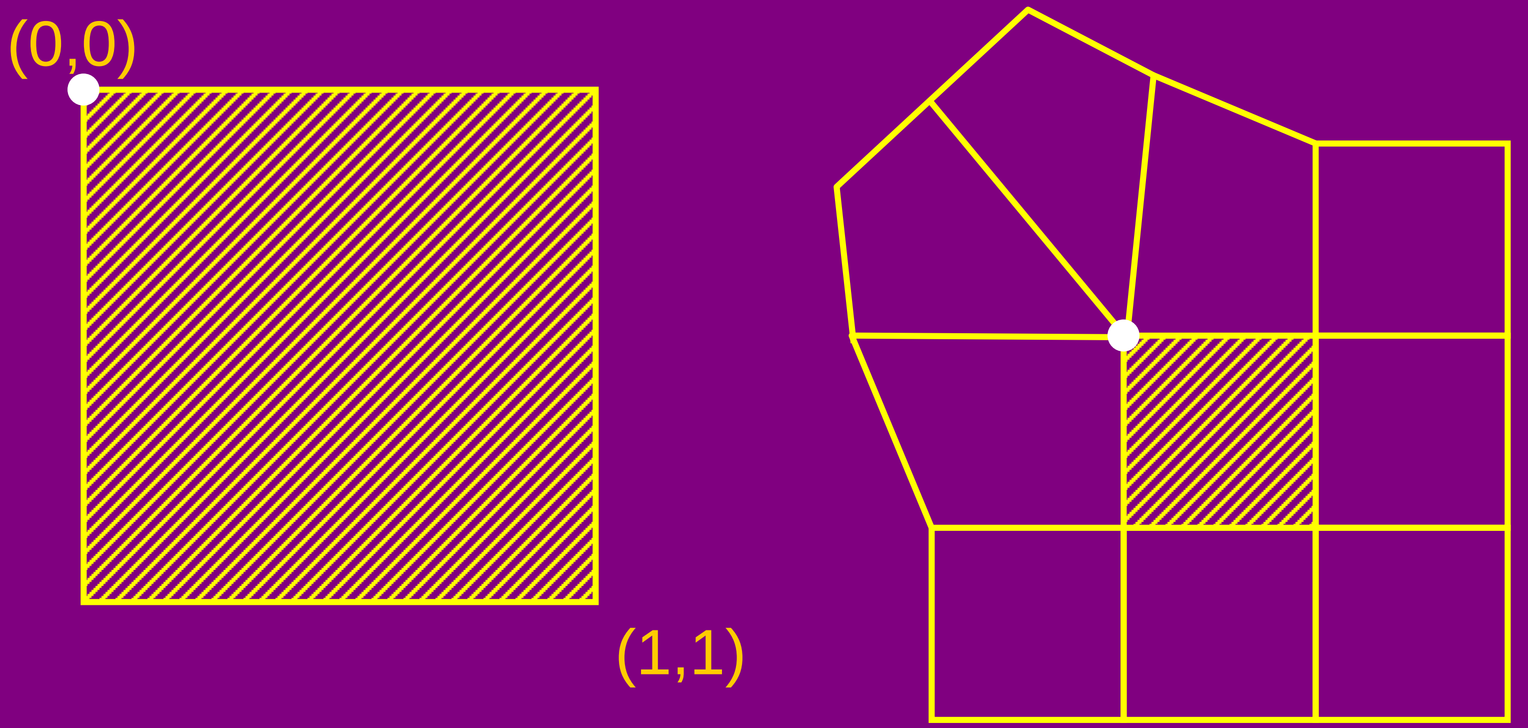


Gauss-Legendre Points

Exact Evaluation



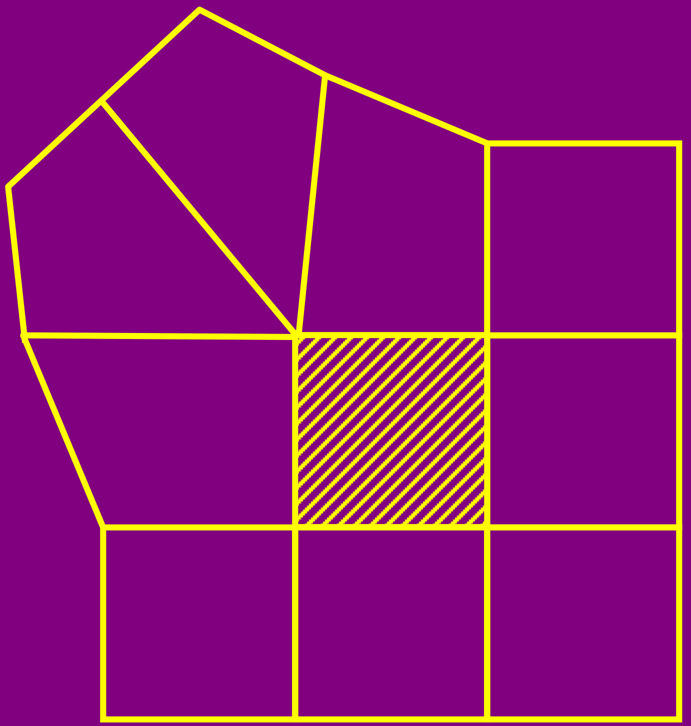
Exact Evaluation



$$(u,v) \longrightarrow f(u,v)$$

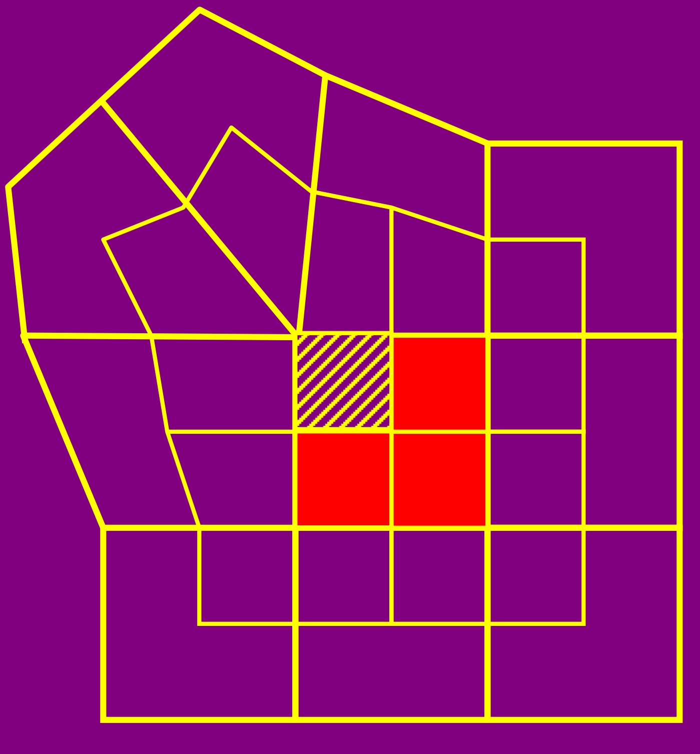
Idea #1

Subdivide once

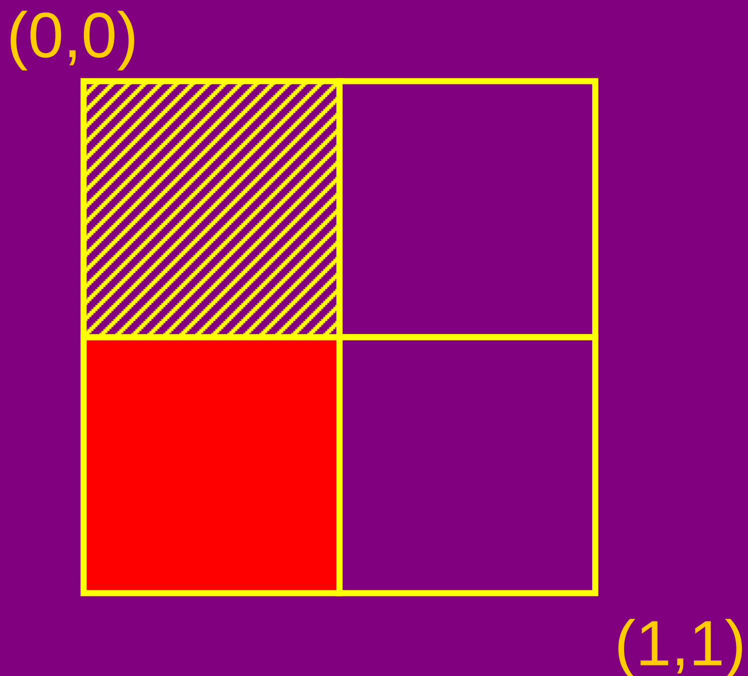


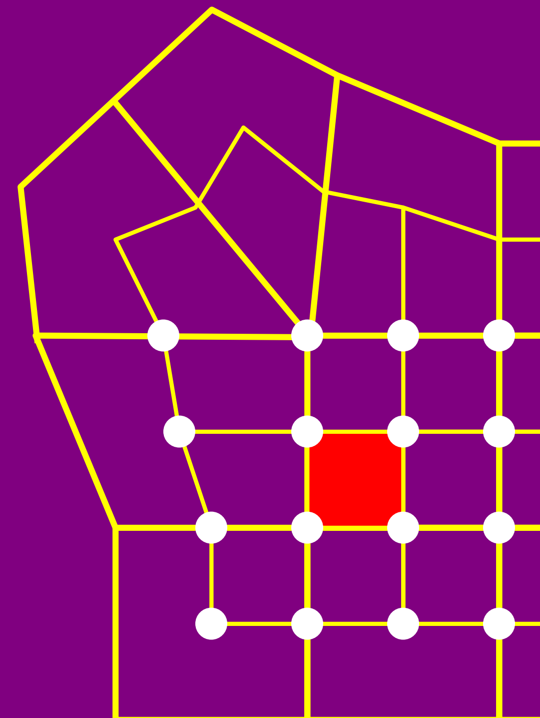
Idea #1

Subdivide once

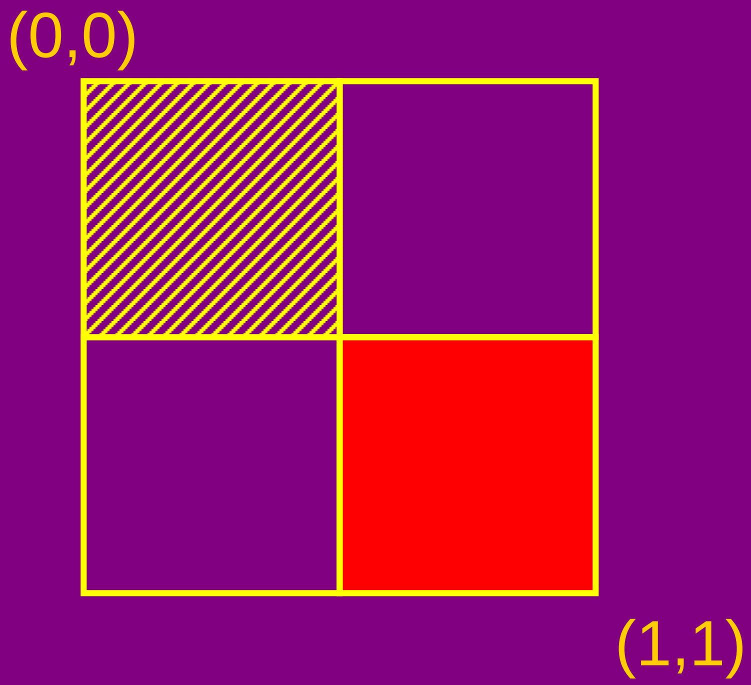


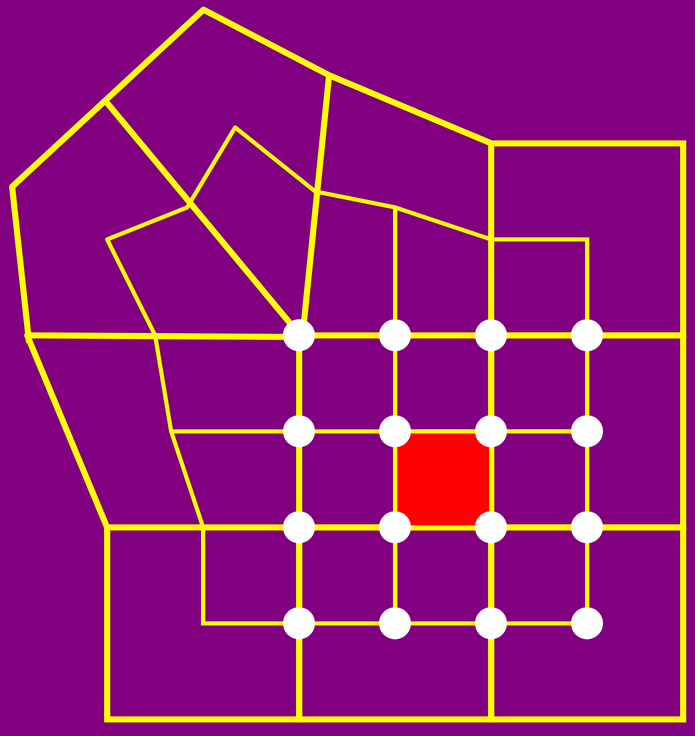
Idea #1



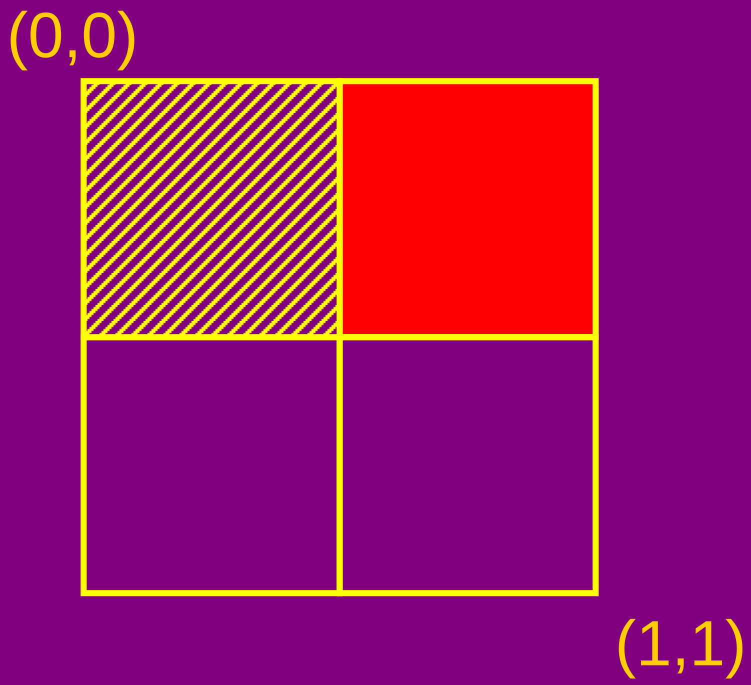


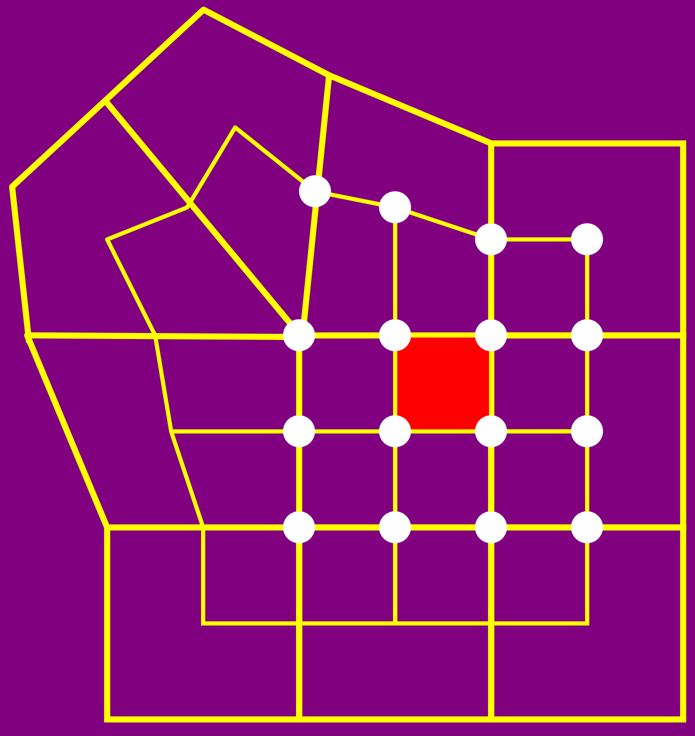
Idea #1



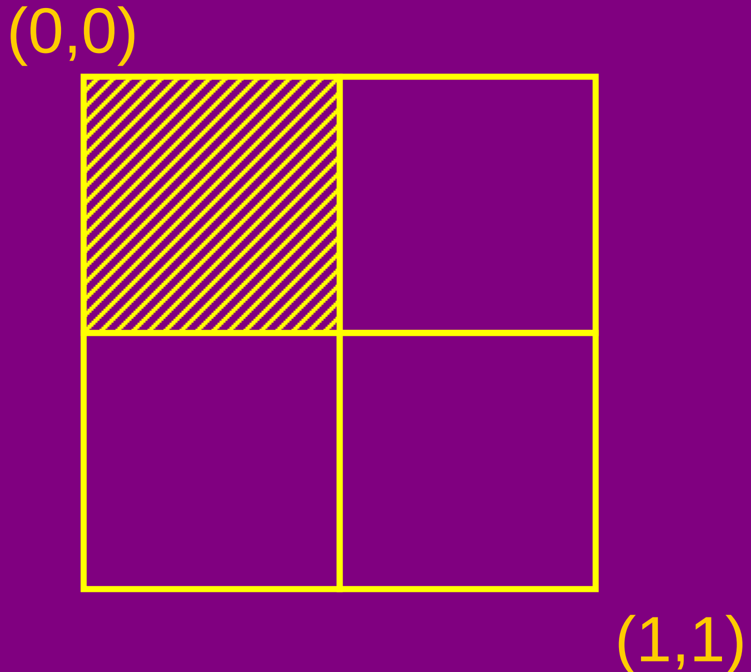


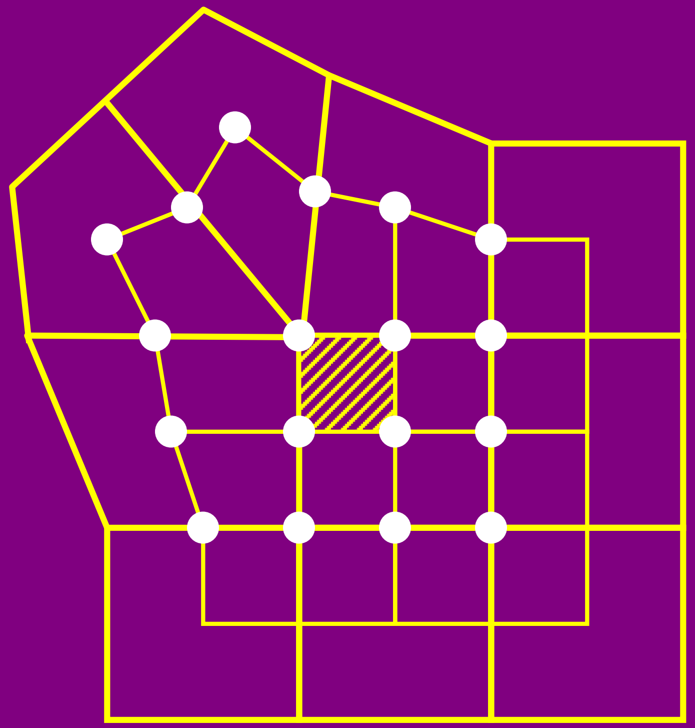
Idea #1



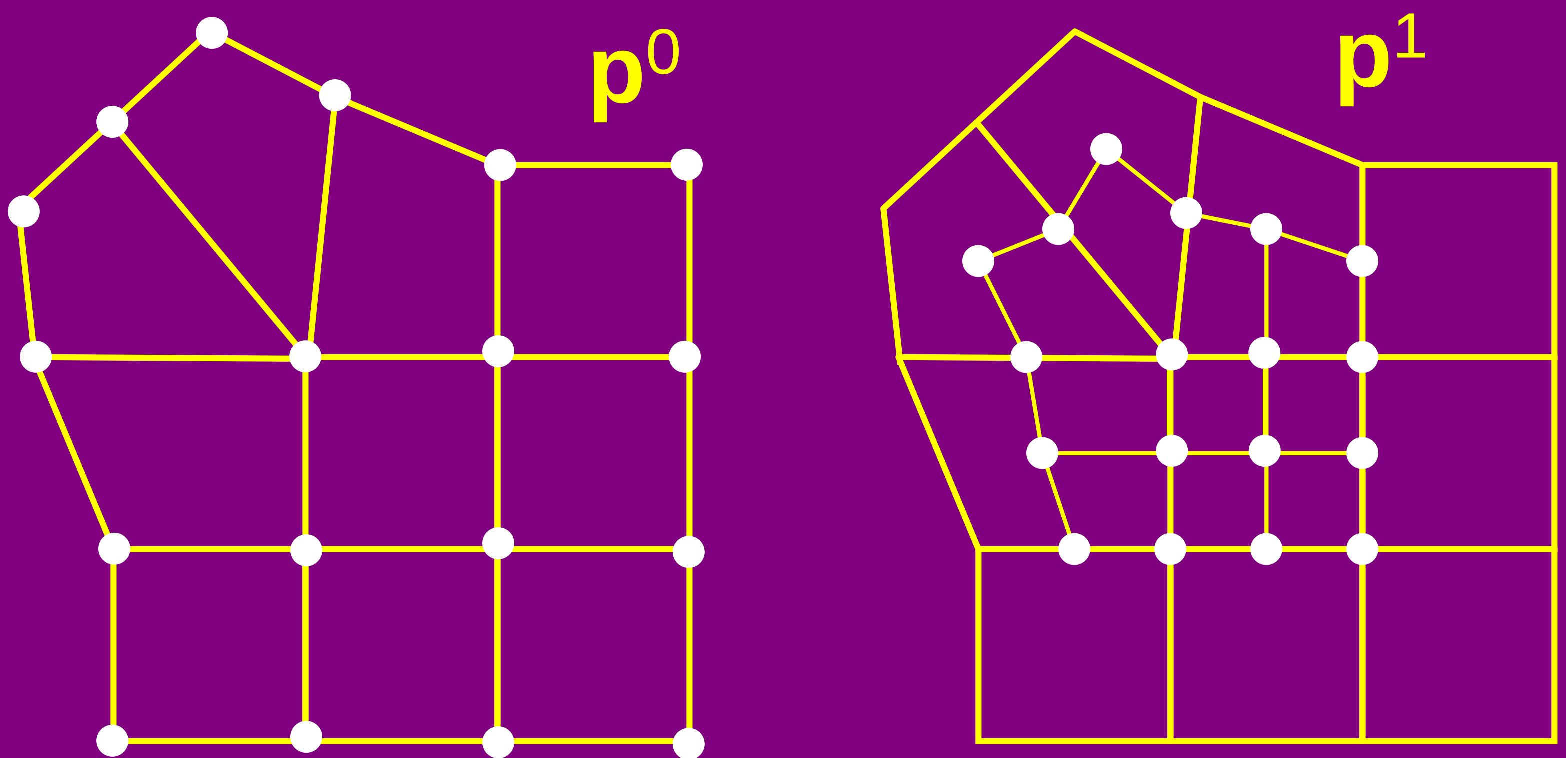


Idea #1



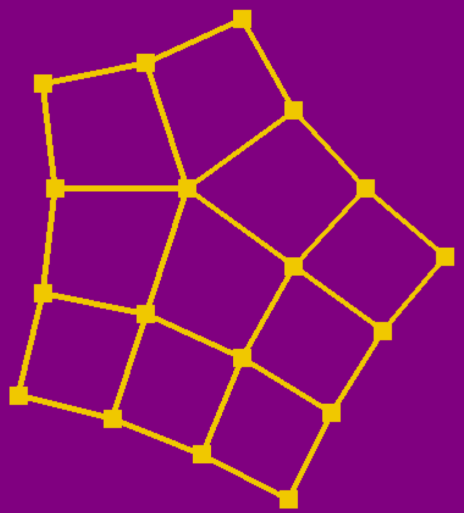


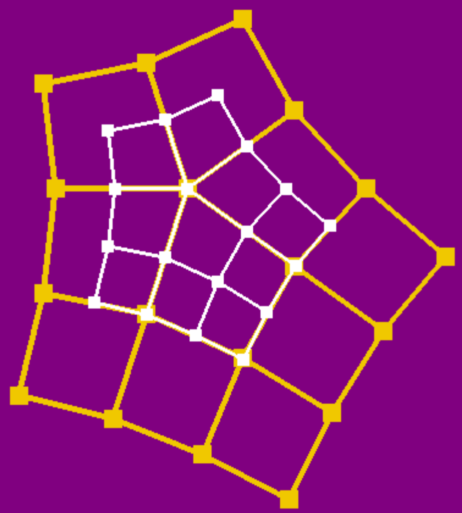
Idea #2



Subdivision almost like scaling: $\mathbf{p}^1 = \mathbf{S}\,\mathbf{p}^0$

Idea #2





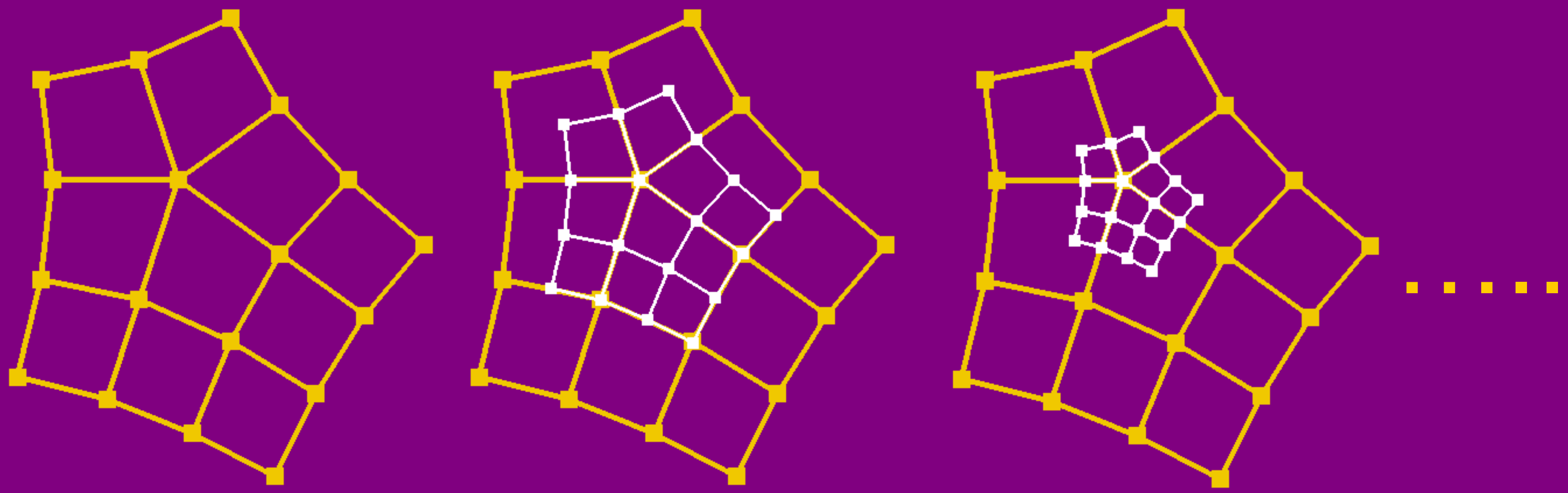
eigenvector

x λ

$\lambda = 0.549988\ldots$ eigenvalue

Subdivision is exactly scaling: $\mathbf{x}^1 = \lambda\, \mathbf{x}^0$

Idea #2

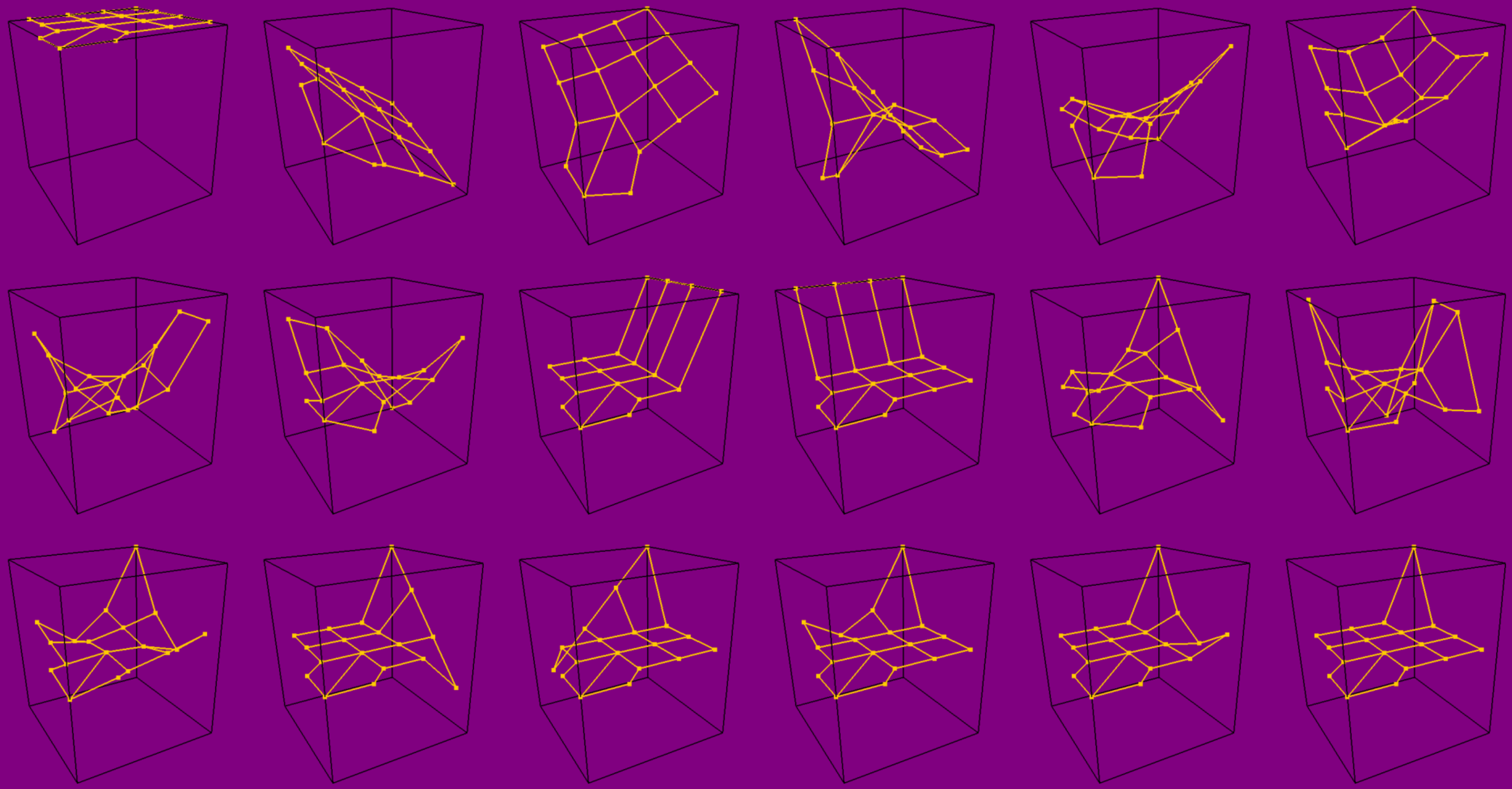


$\times \lambda$ $\qquad$ $\times \lambda^2$ $\qquad$ $\times \lambda^n$

$\lambda = 0.549988...$

Subdivision is **exactly** scaling: $\mathbf{x}^2 = \lambda\, \mathbf{x}^1 = \lambda^2\, \mathbf{x}^0$

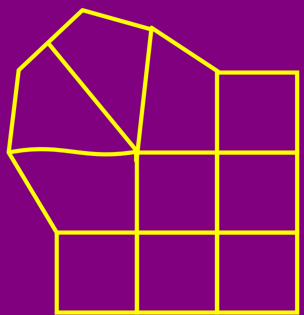
Idea #2



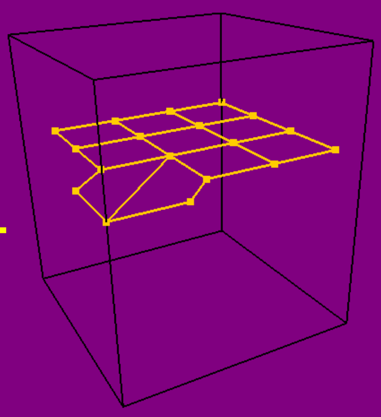
18 Eigenvectors of Catmull-Clark for N=5

Idea #2

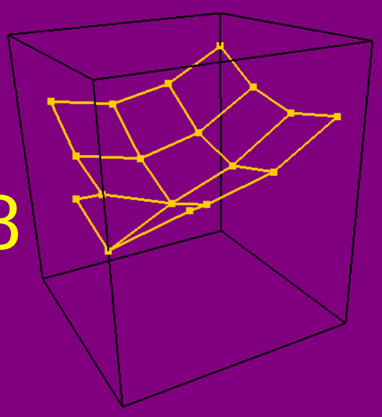
Any control mesh is a linear combination of the eigenvectors:



$= \alpha_1$



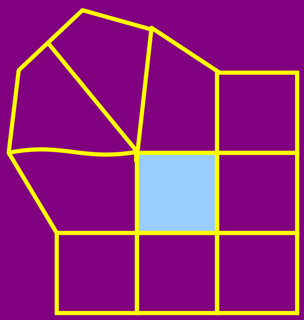
$+ \ldots + \alpha_{18}$



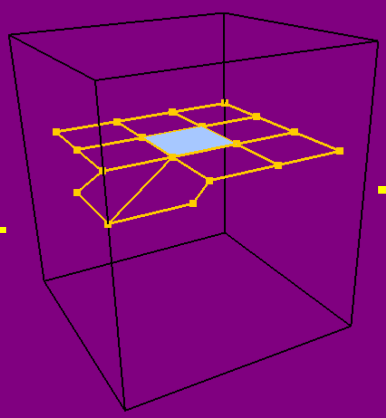
Eigenvectors form a basis

Idea #2

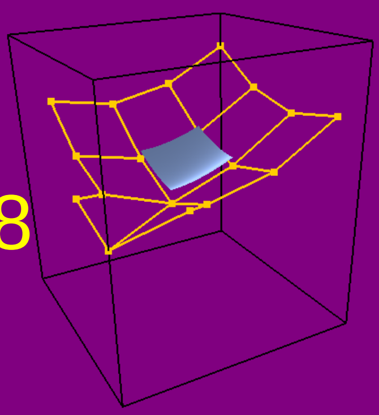
Only have to consider the eigenvectors



$= \alpha_1$

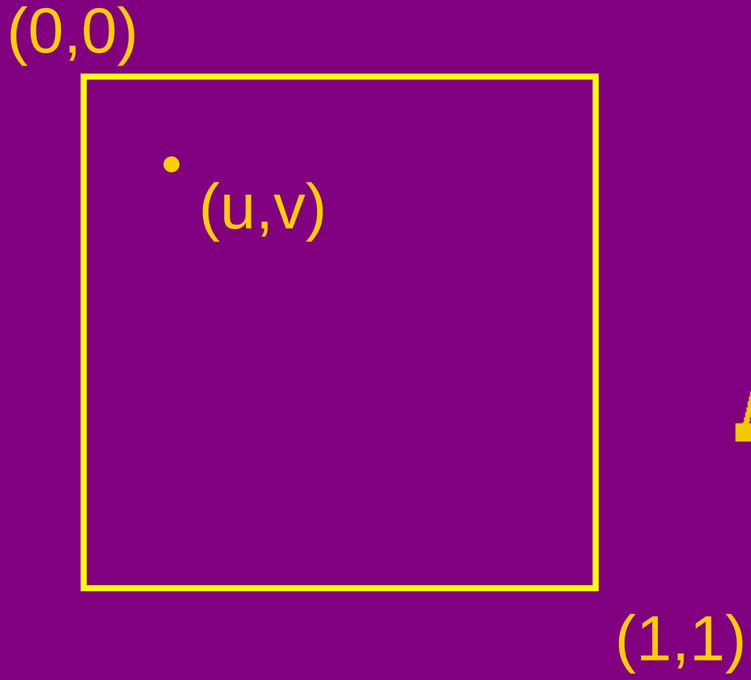


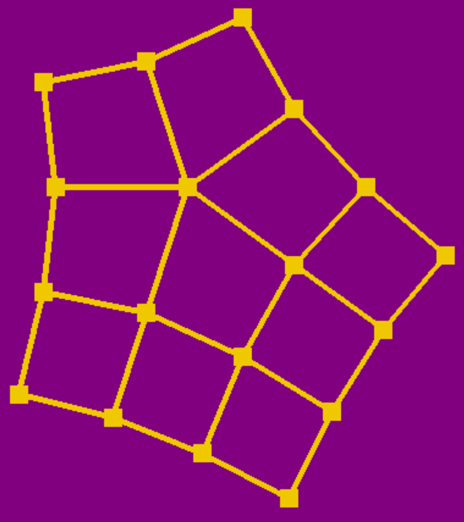
$+ \ldots + \alpha_{18}$



$$S(u,v) = \alpha_1 \, \phi_1(u,v) + \ldots + \alpha_{18} \, \phi_{18}(u,v)$$

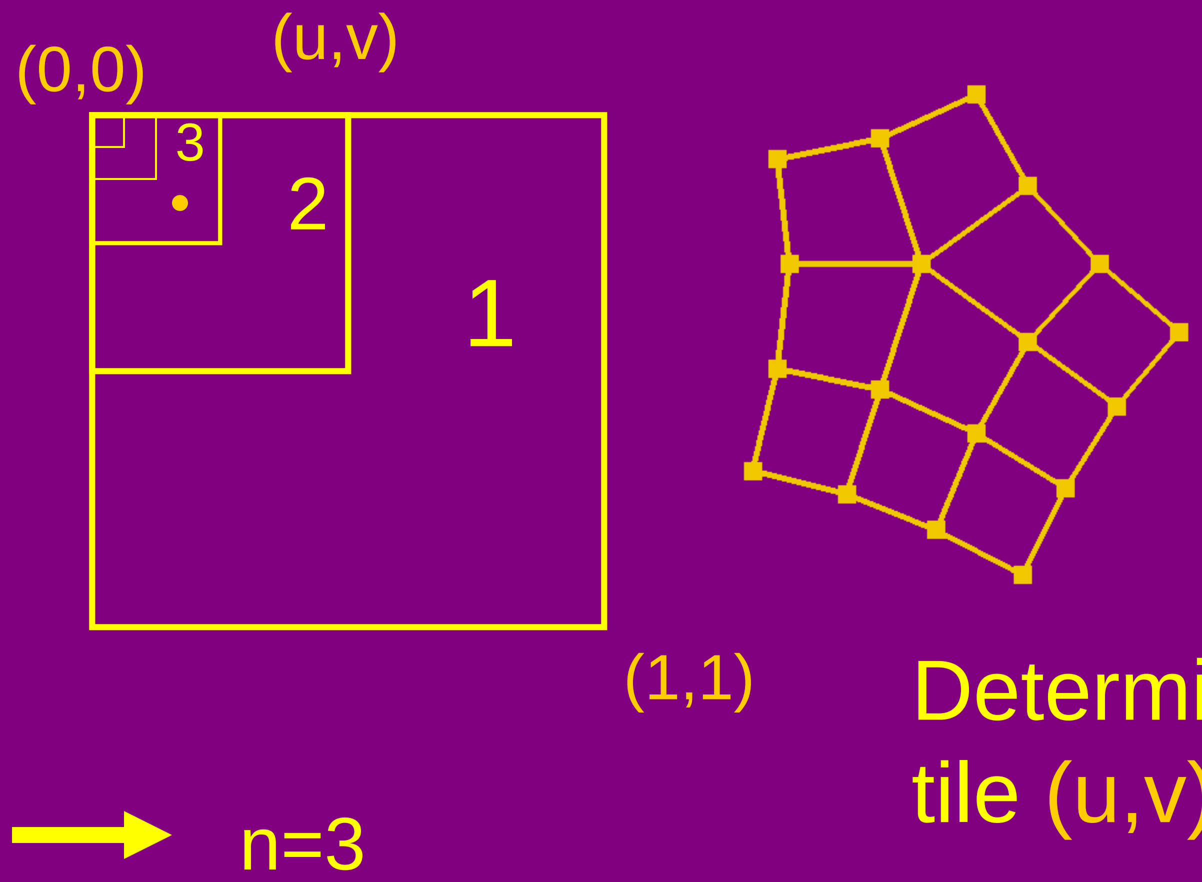
Evaluation





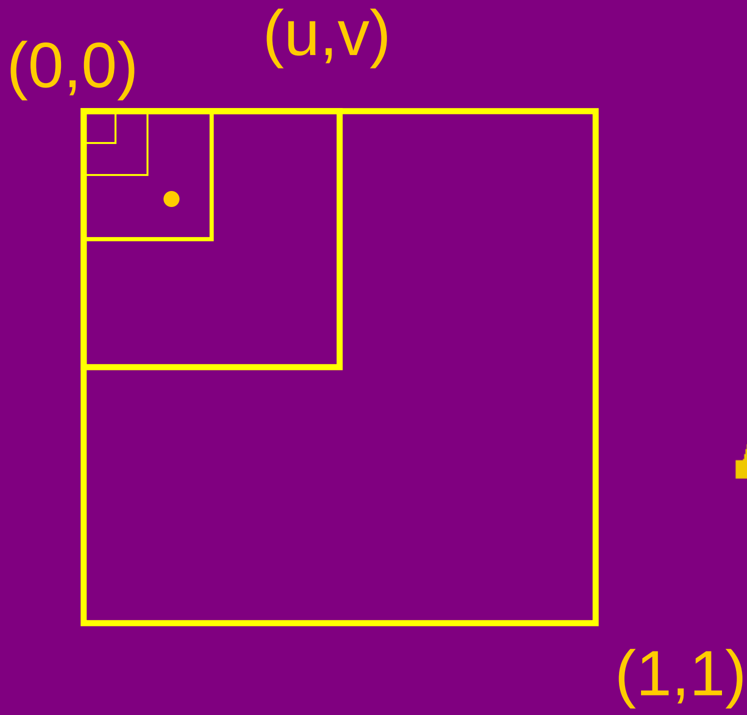
Want to evaluate $\phi_i(u,v)$ at a particular (u,v) location

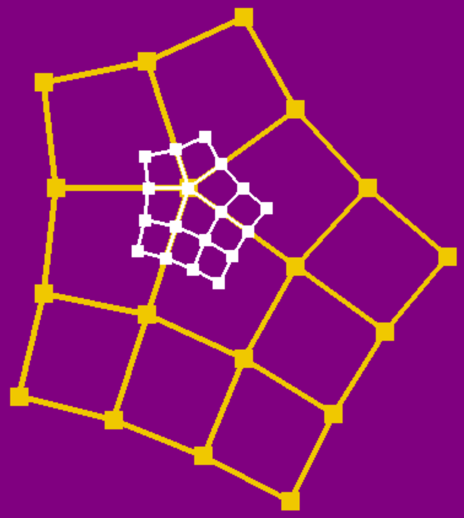
Evaluation



Determine in which tile (u,v) lies

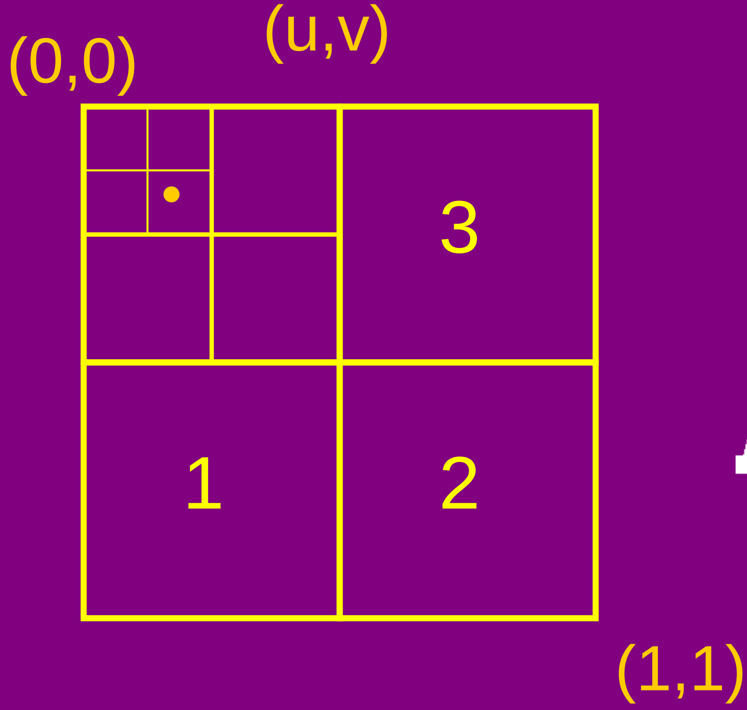
Evaluation

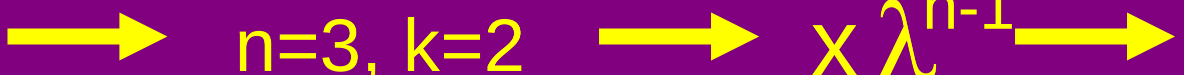




$\times \lambda^{n-1}$

Evaluation

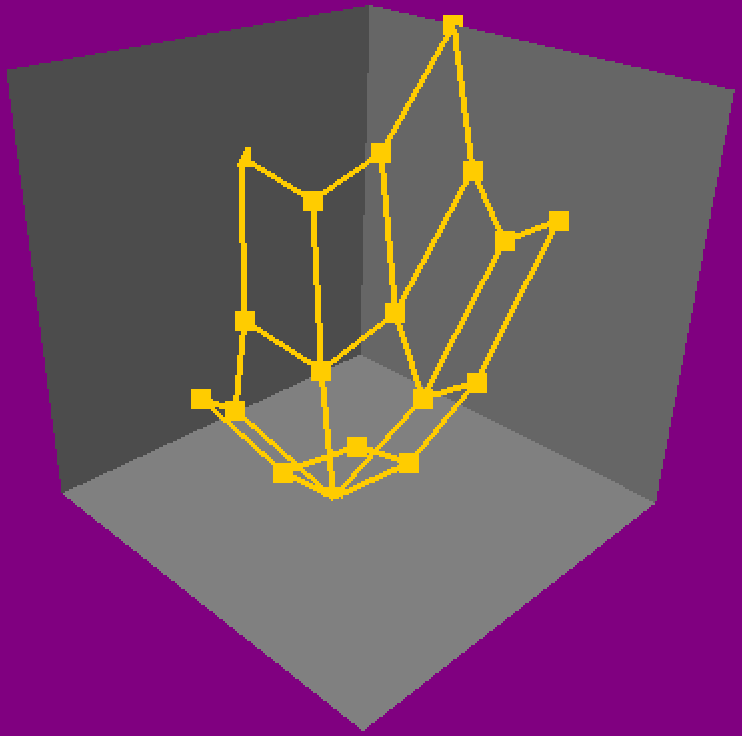


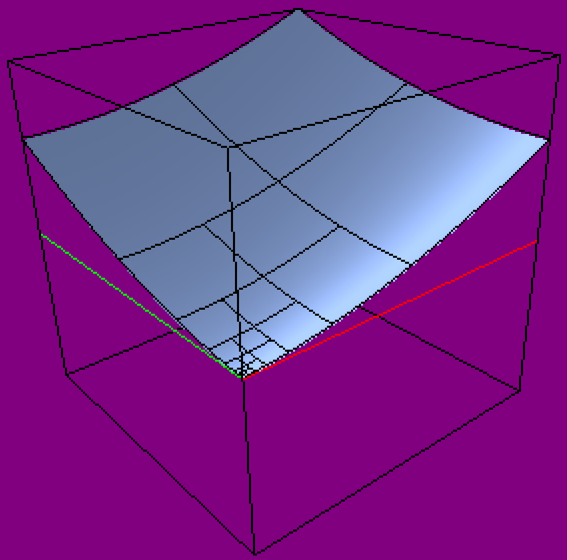


$\times \lambda^{n-1}$

$\phi_i(u,v)$

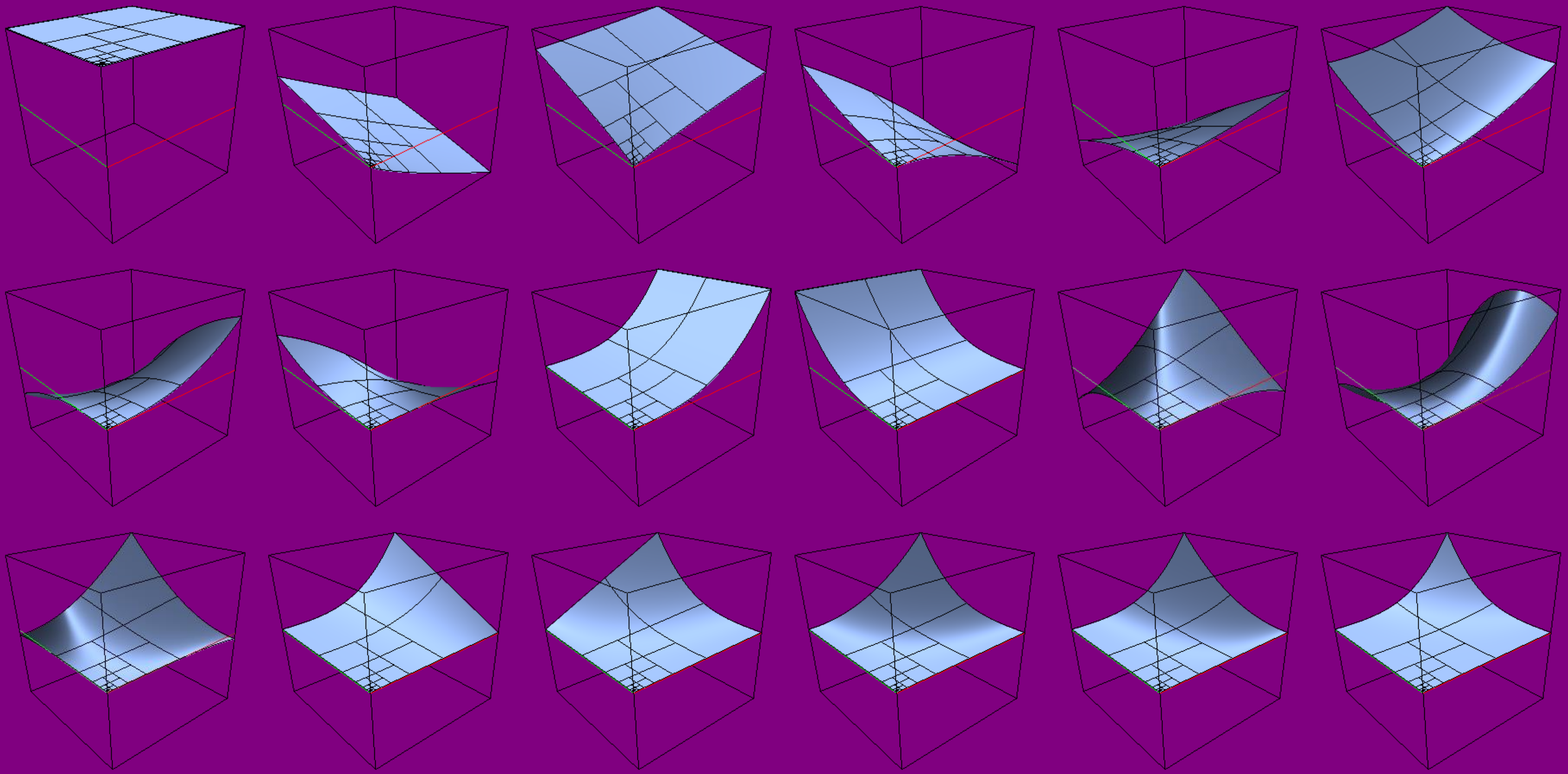
New Basis Functions





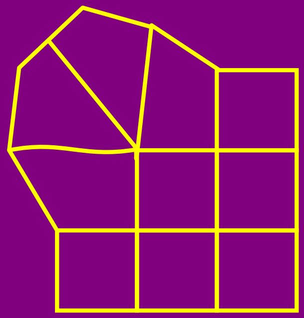
$\phi_i\,(u,v)$

New Basis Functions



18 Basis functions for Catmull-Clark with N=5

Exact Evaluation



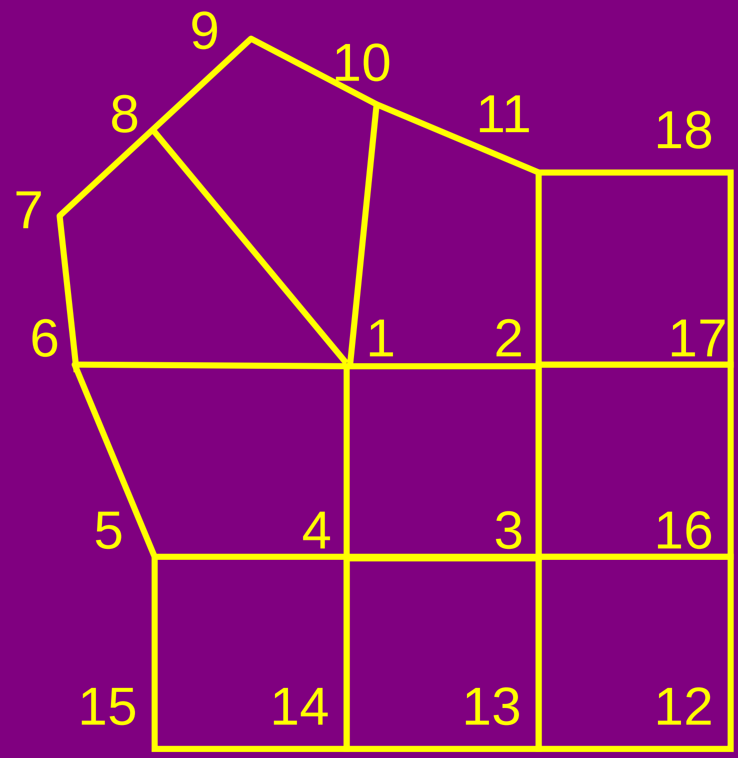
C_i : control vertices

α_i : linear combination of the CVs

$$f(u,v) = \sum_{i=1}^{K} \alpha_i \, \phi_i(u,v)$$

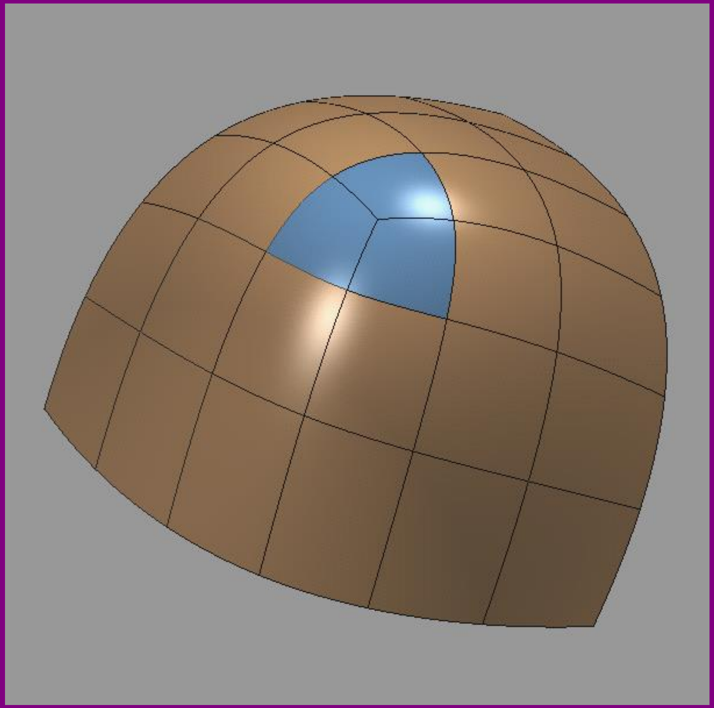
On the CDROM

Eigenstructures for N < 50

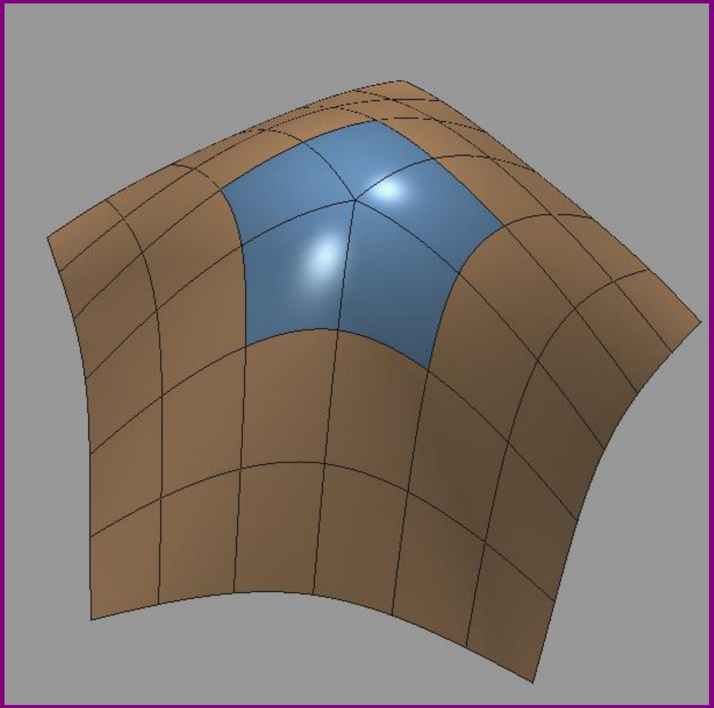


Make sure to get the ordering right

Results

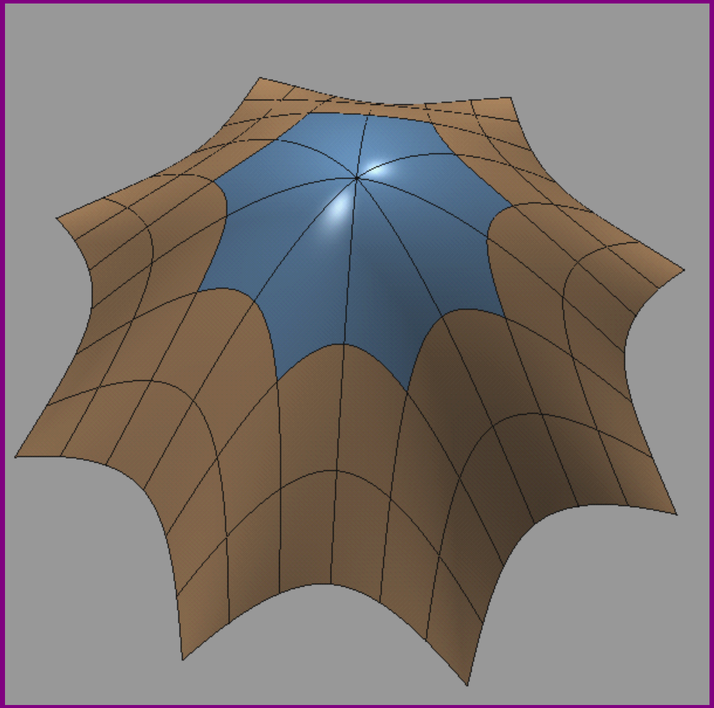


N=3

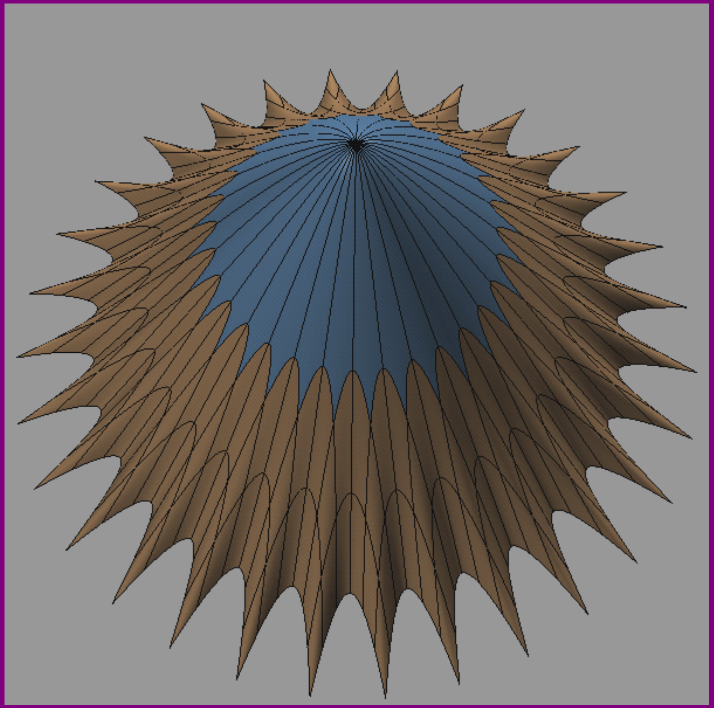


N=5

Results

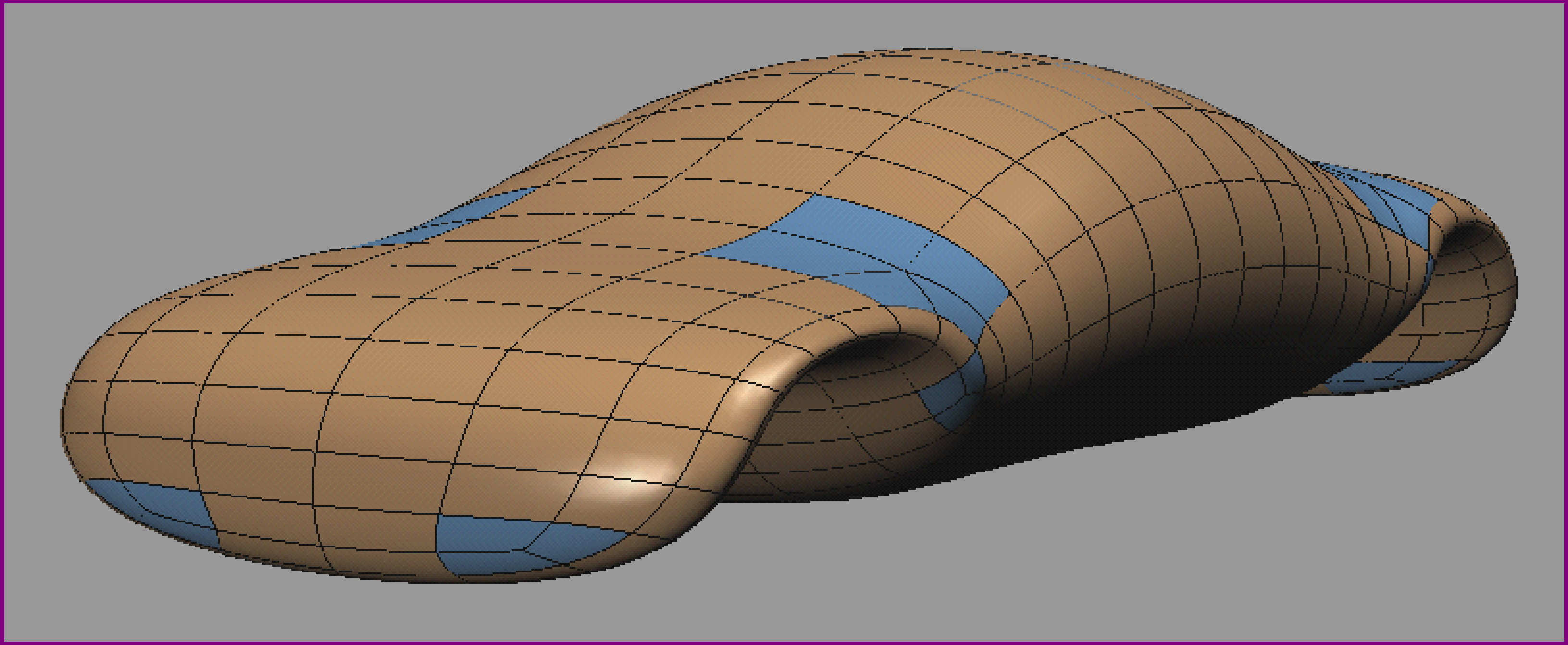


N=8

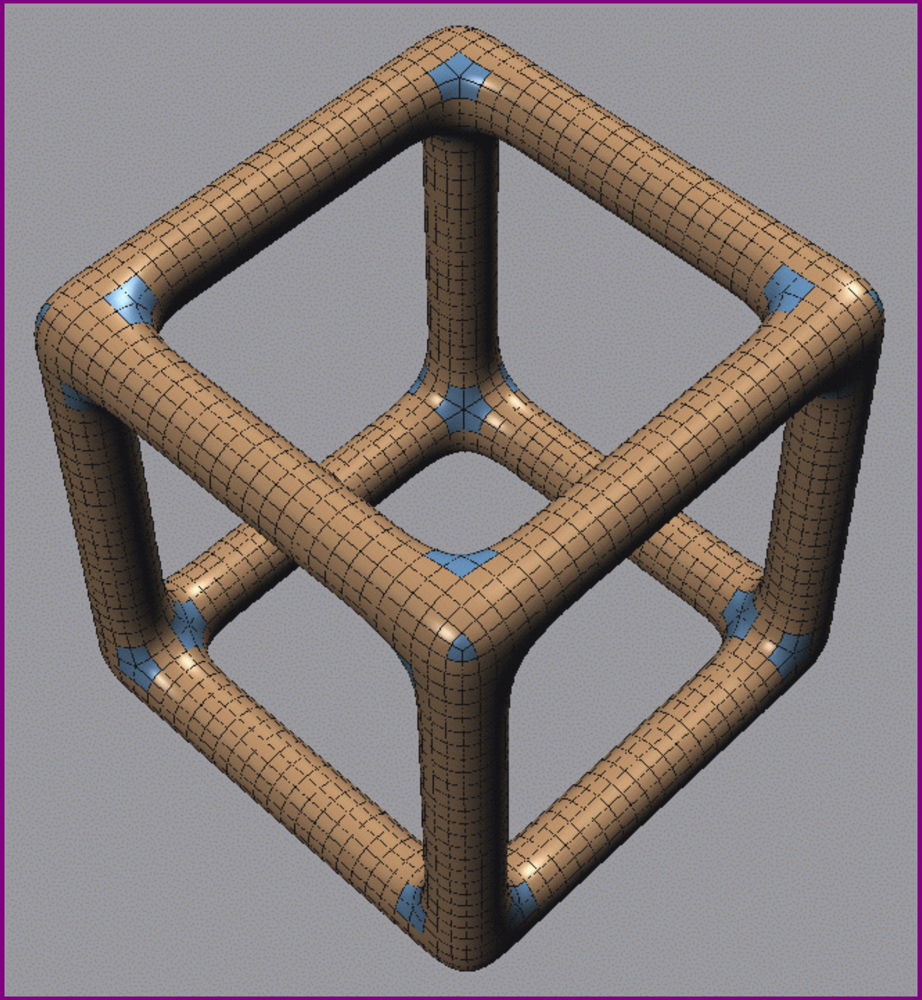


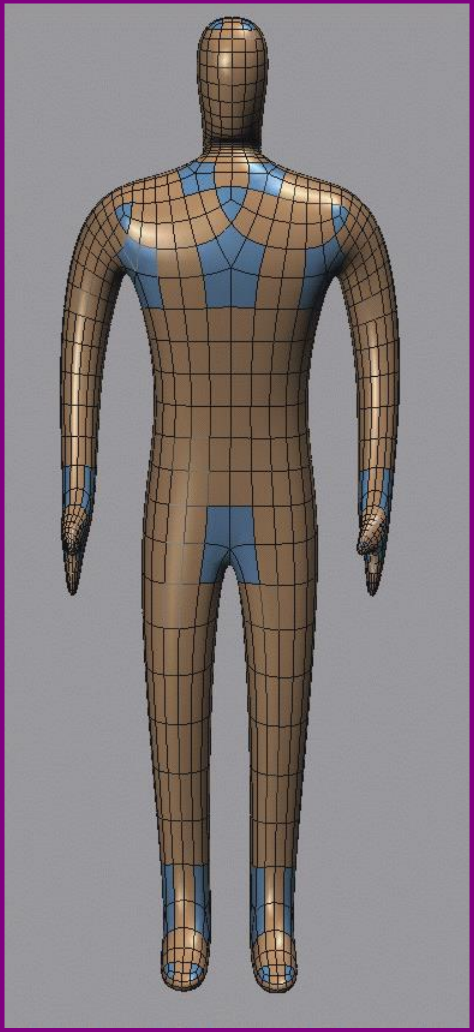
N=30

Results

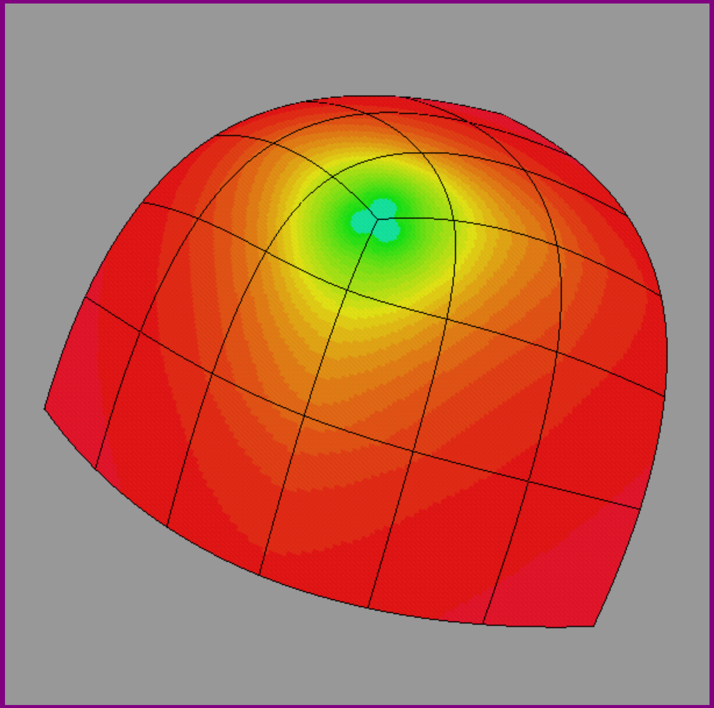


Results

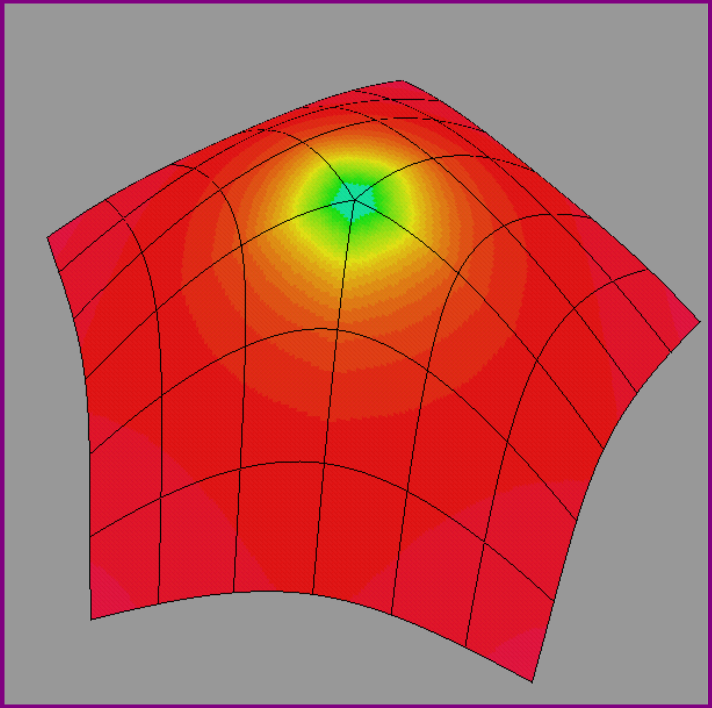




Results

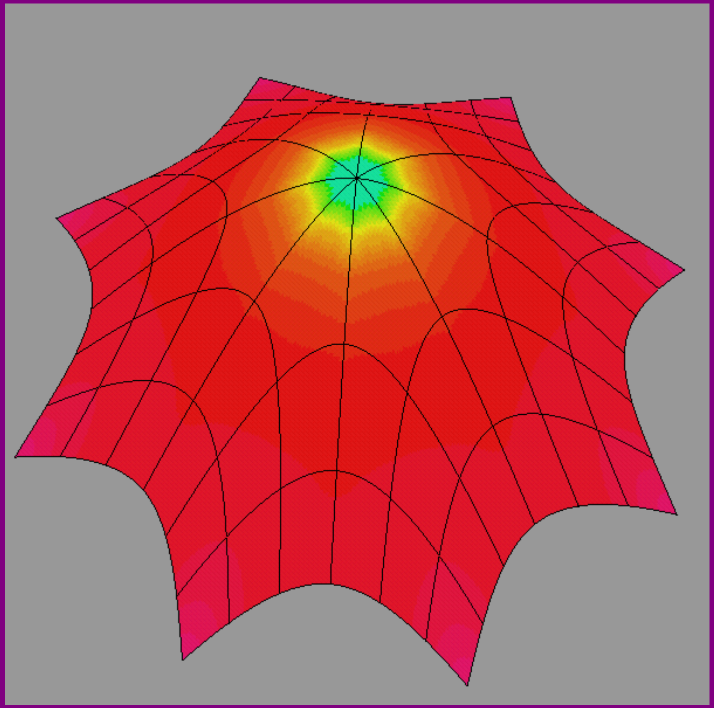


N=3

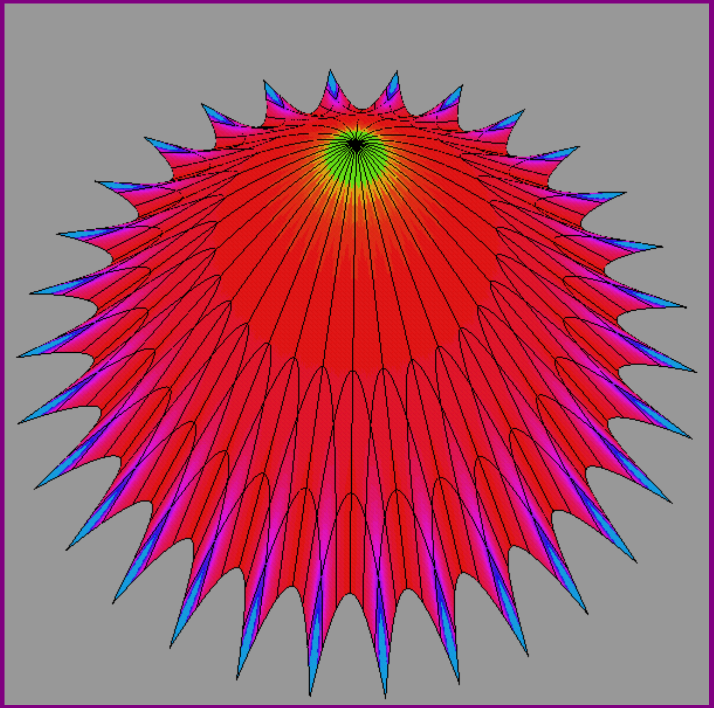


N=5

Results



N=8



N=30

Other Schemes

Also works for:

- Loop triangular surfaces (Jordan for N=3)
- Doo-Sabin (Jordan)
- Other polynomial and stationary schemes
- Creases

Conclusions

Catmull-Clark surfaces = parametric surfaces

Easy to code

Useful in many applications

Future Work

Forward Differences

Non-polynomial

Non-stationary