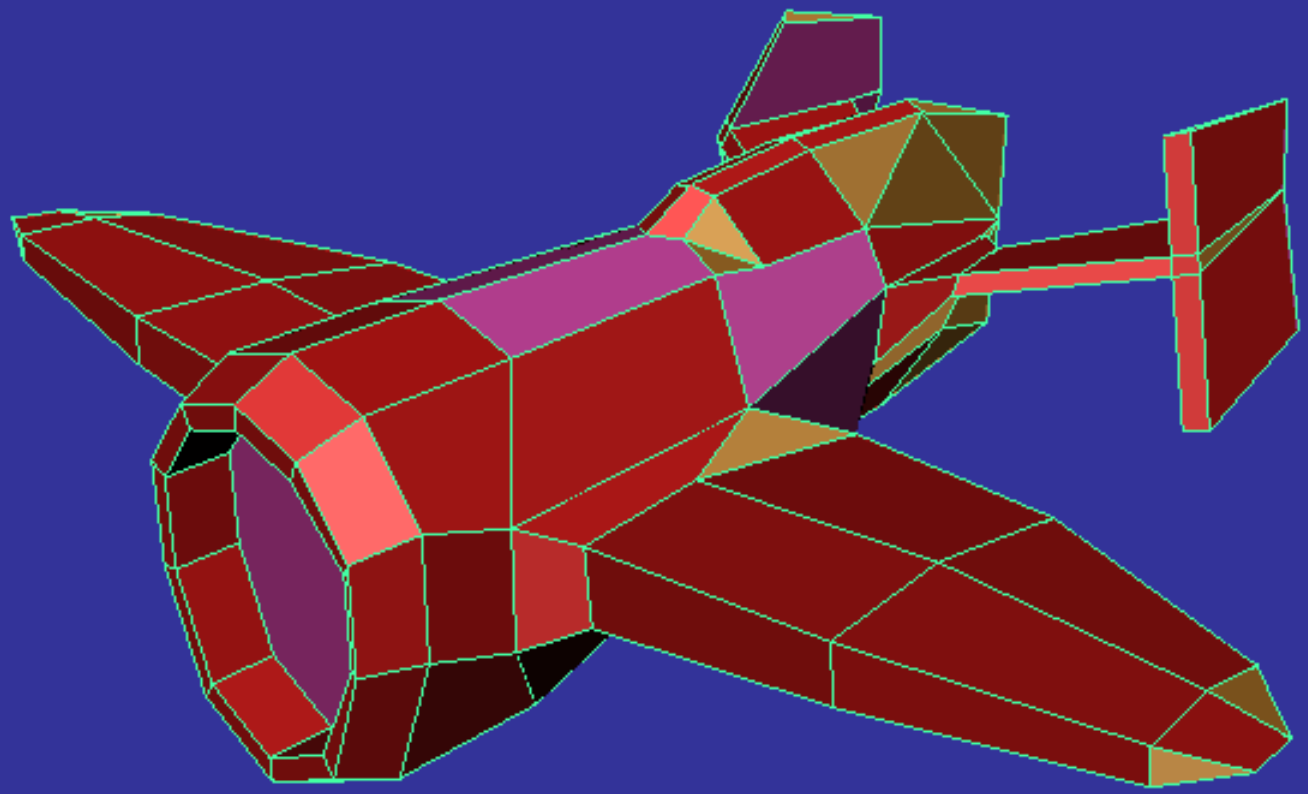
Quad/Triangle Subdivision

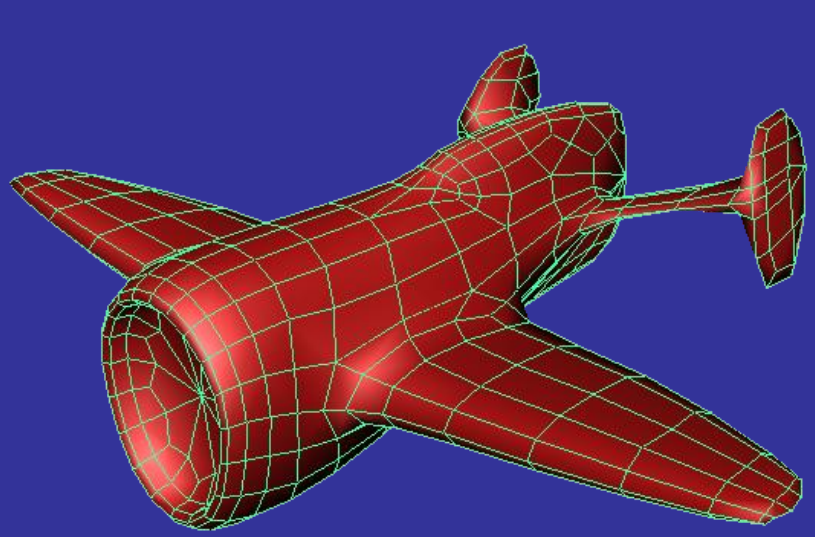
Jos Stam
Alias|wavefront

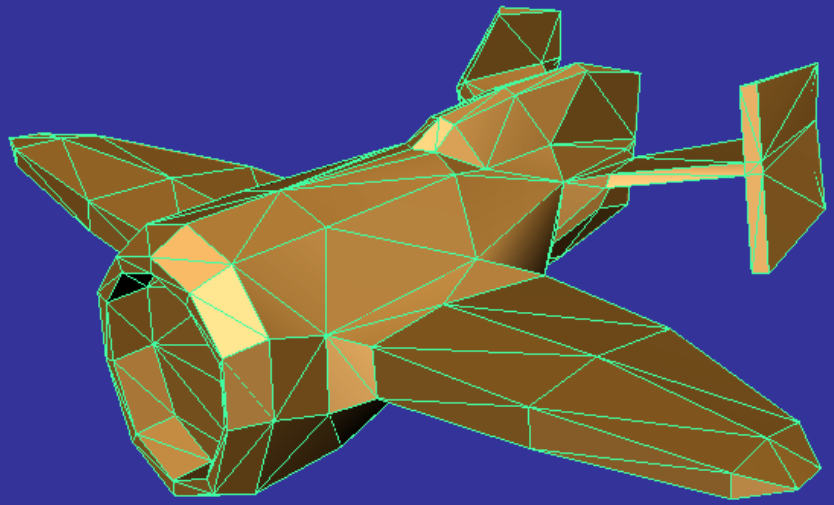
with Charles Loop (Microsoft)

Polygonal Modeling

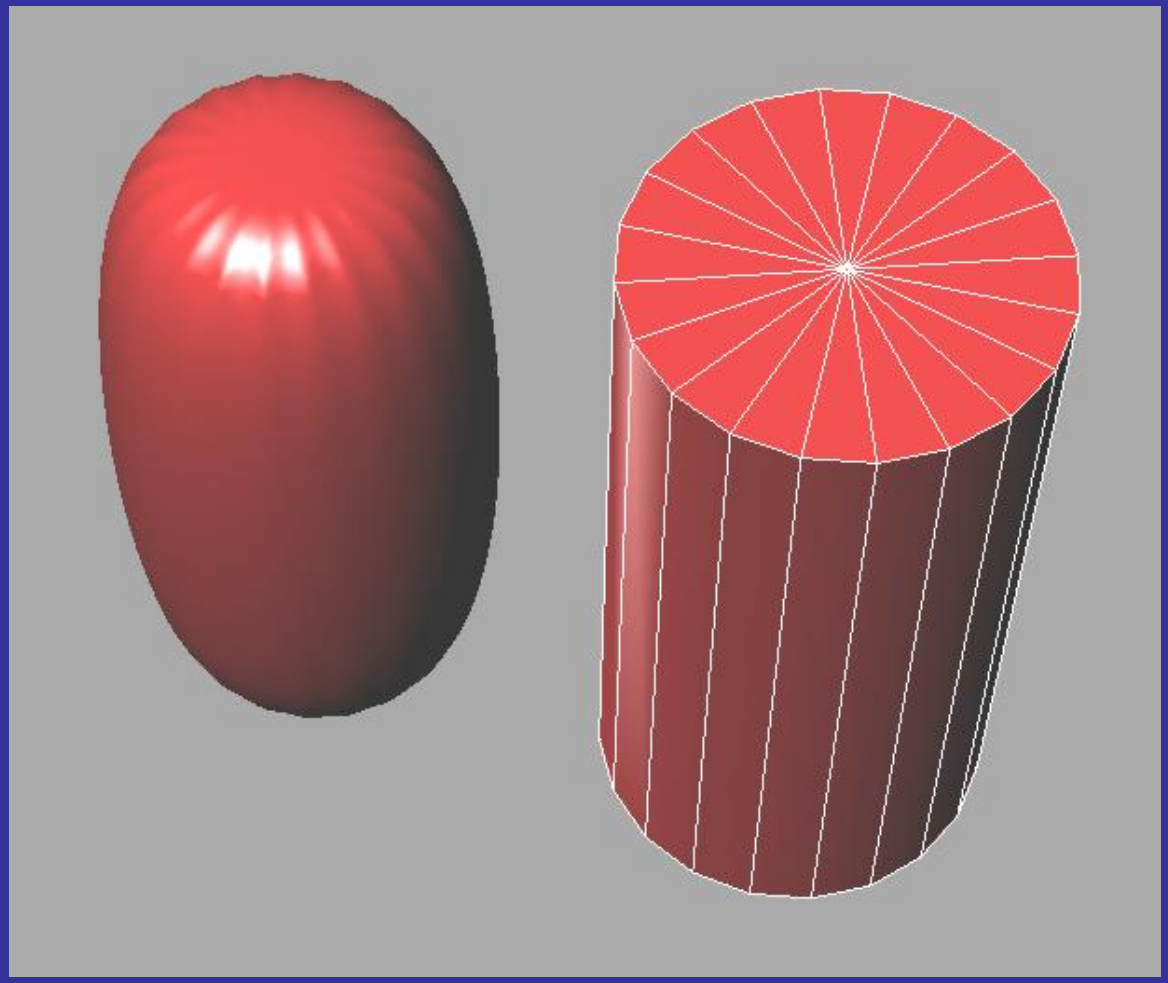


Quads vs Triangles



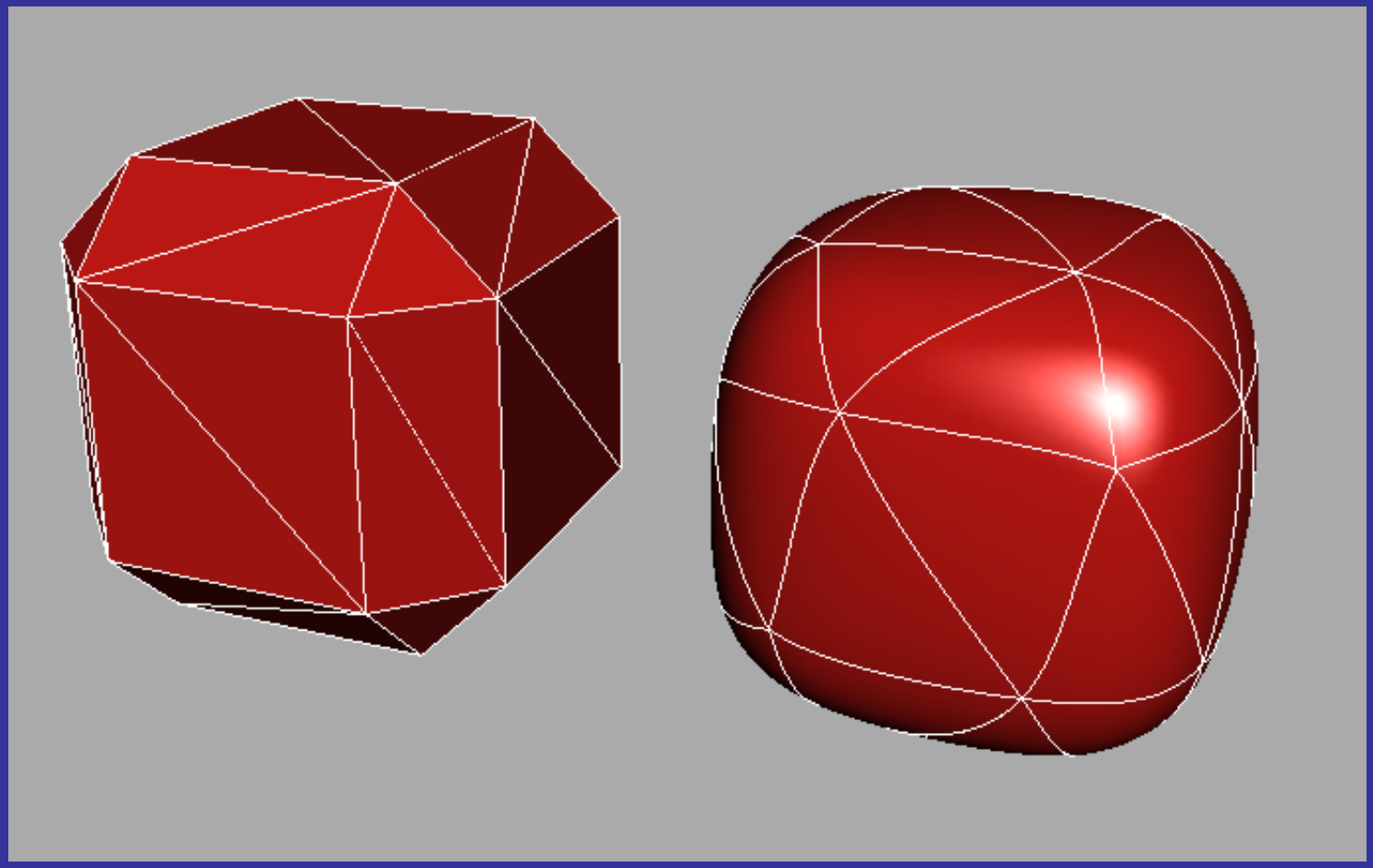


Problems



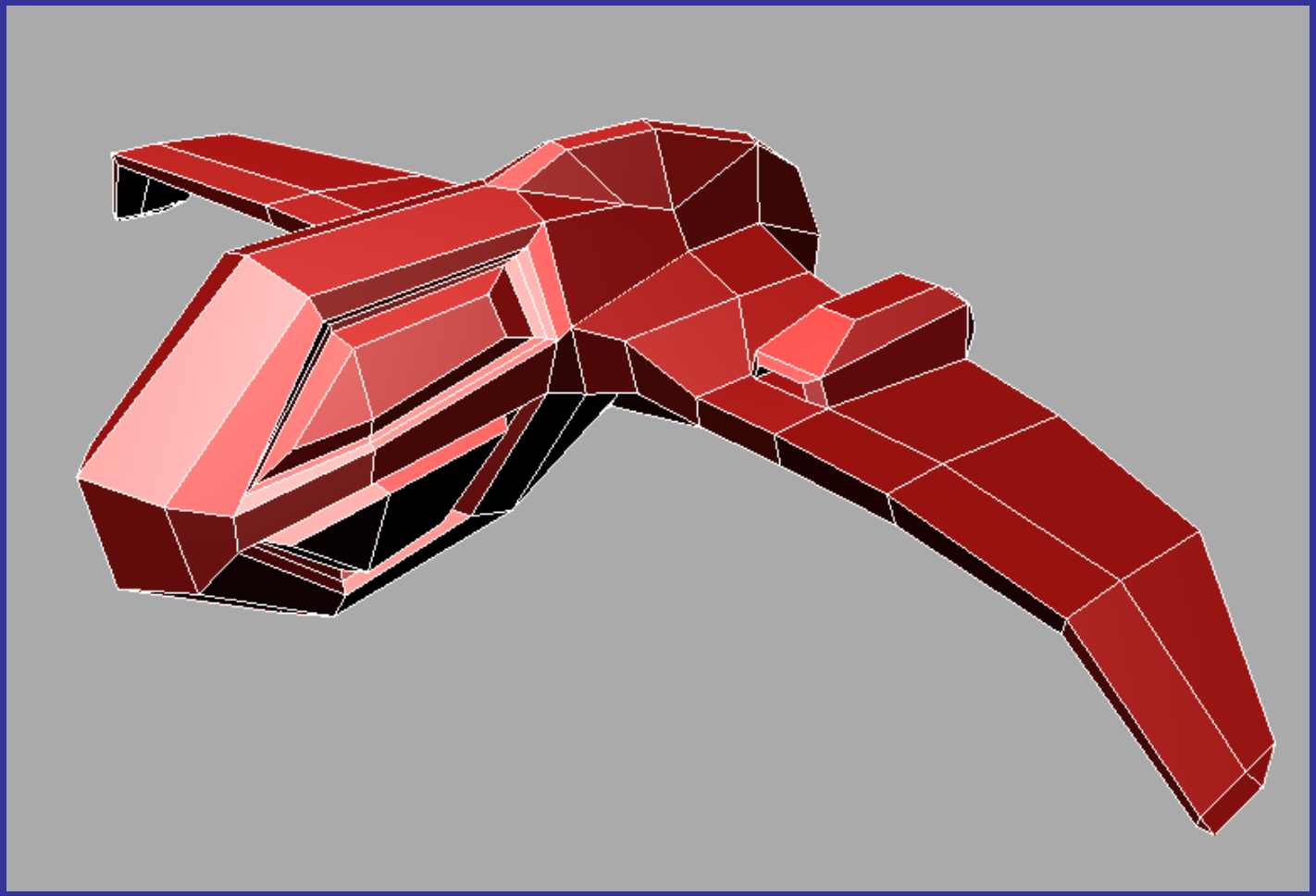
Triangles bad for Catmull-Clark

Problems



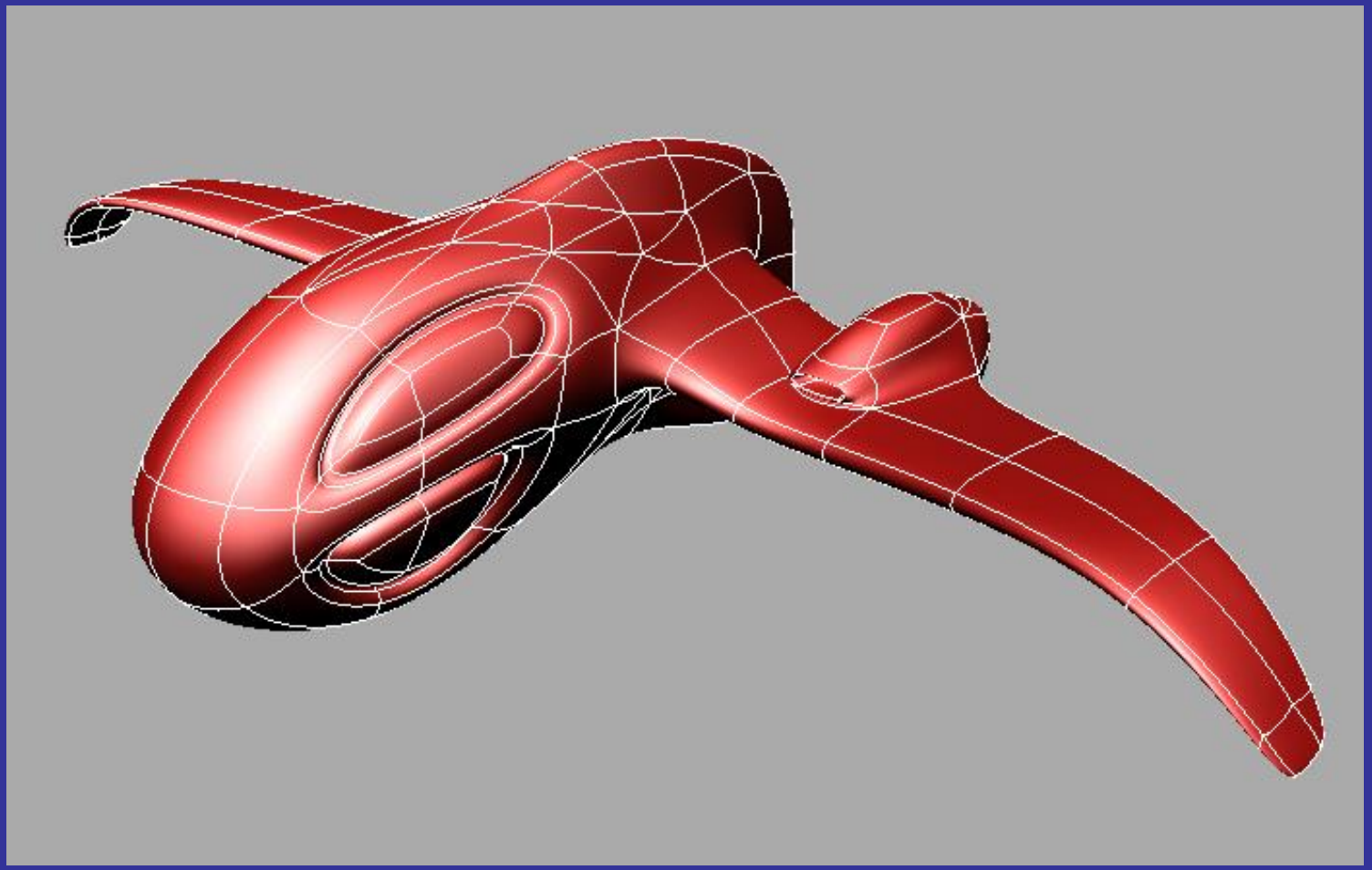
Loop: quad structure lost

Solution



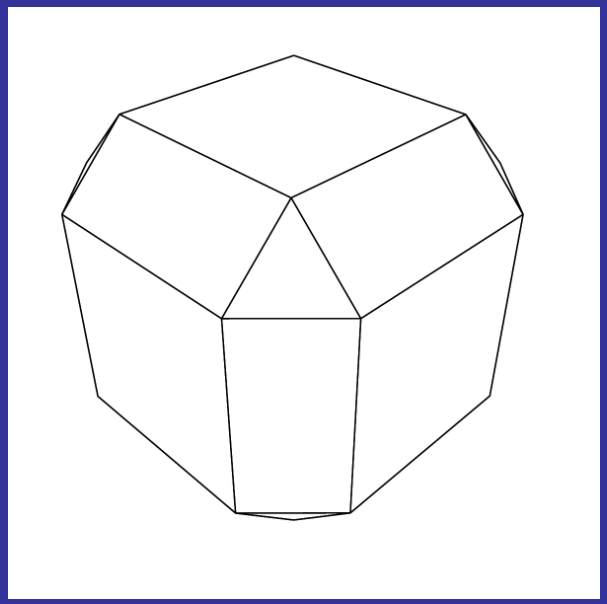
Keep quads and triangles

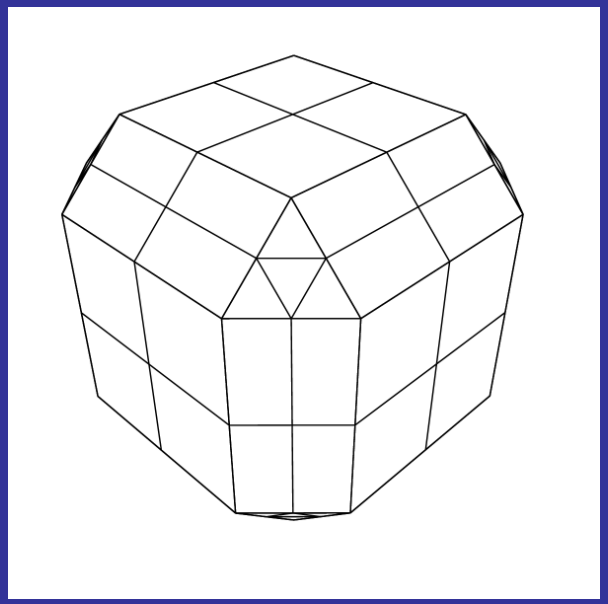
Solution

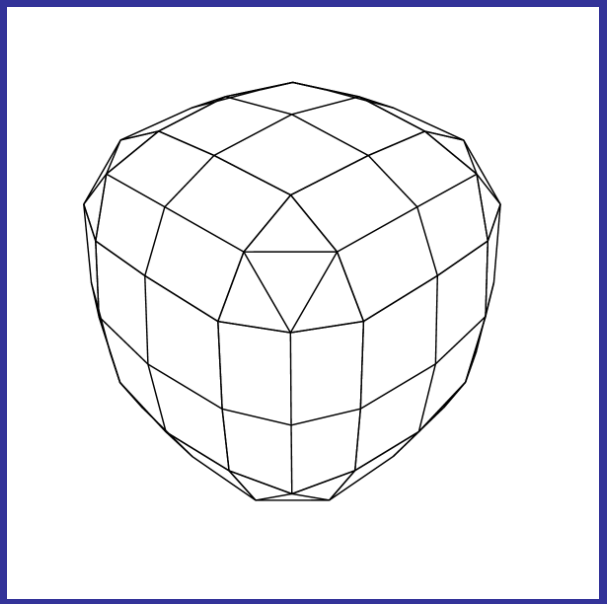


Keep quads and triangles

Algorithm

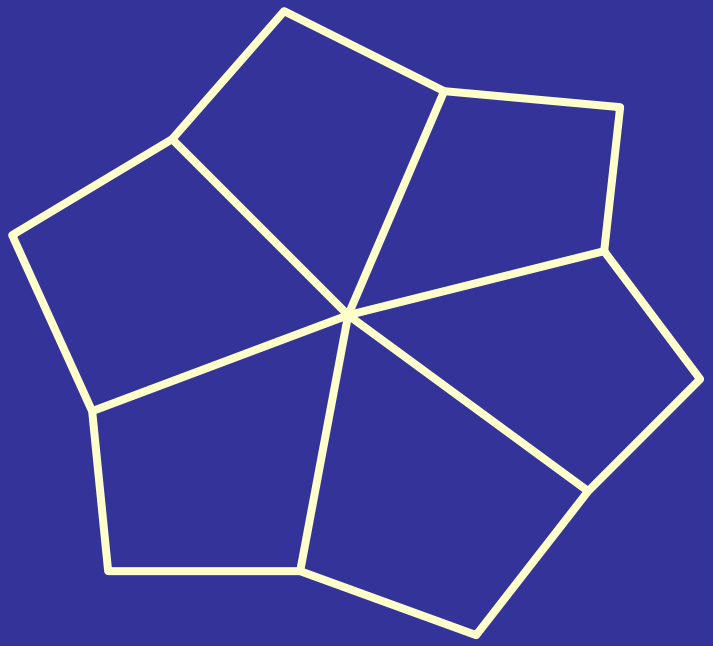






Use split and average

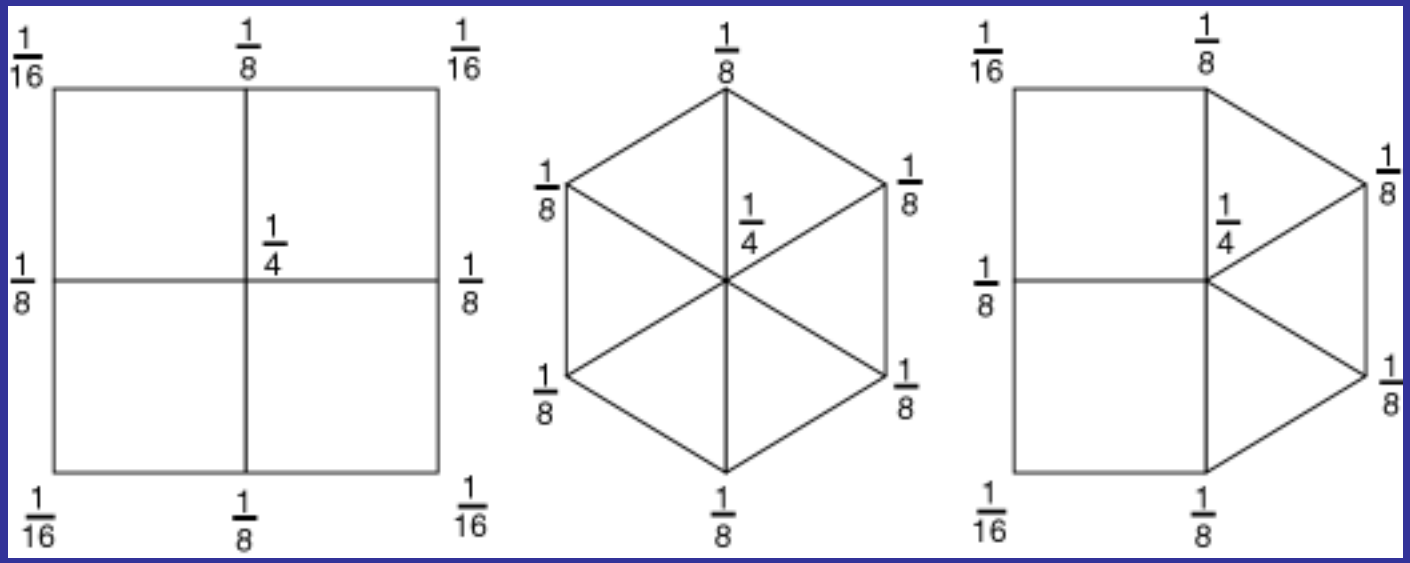
Known Masks



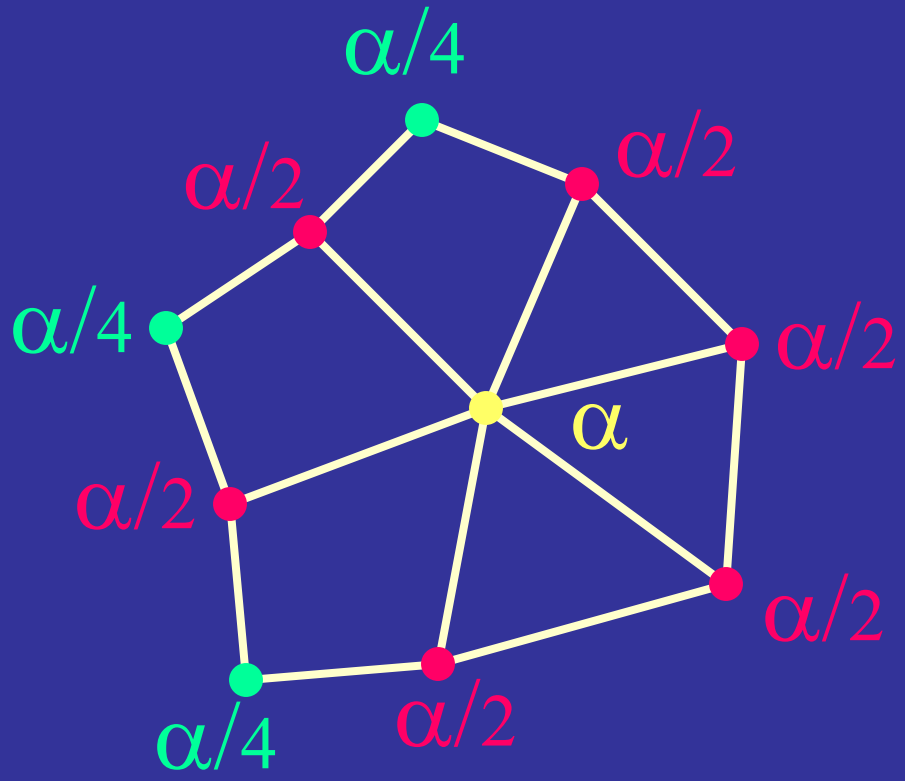
Catmull-Clark

Loop

Regular Quad/Triangle

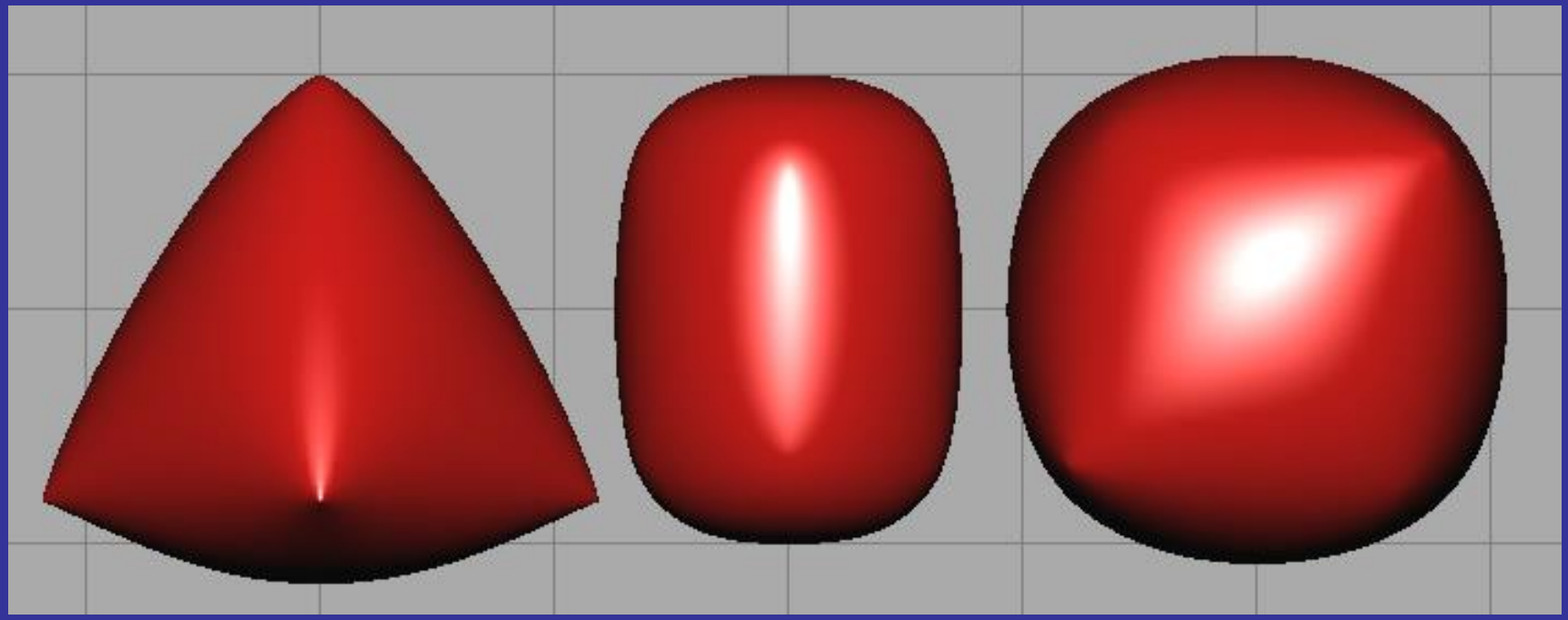


Irregular Quad/Triangle



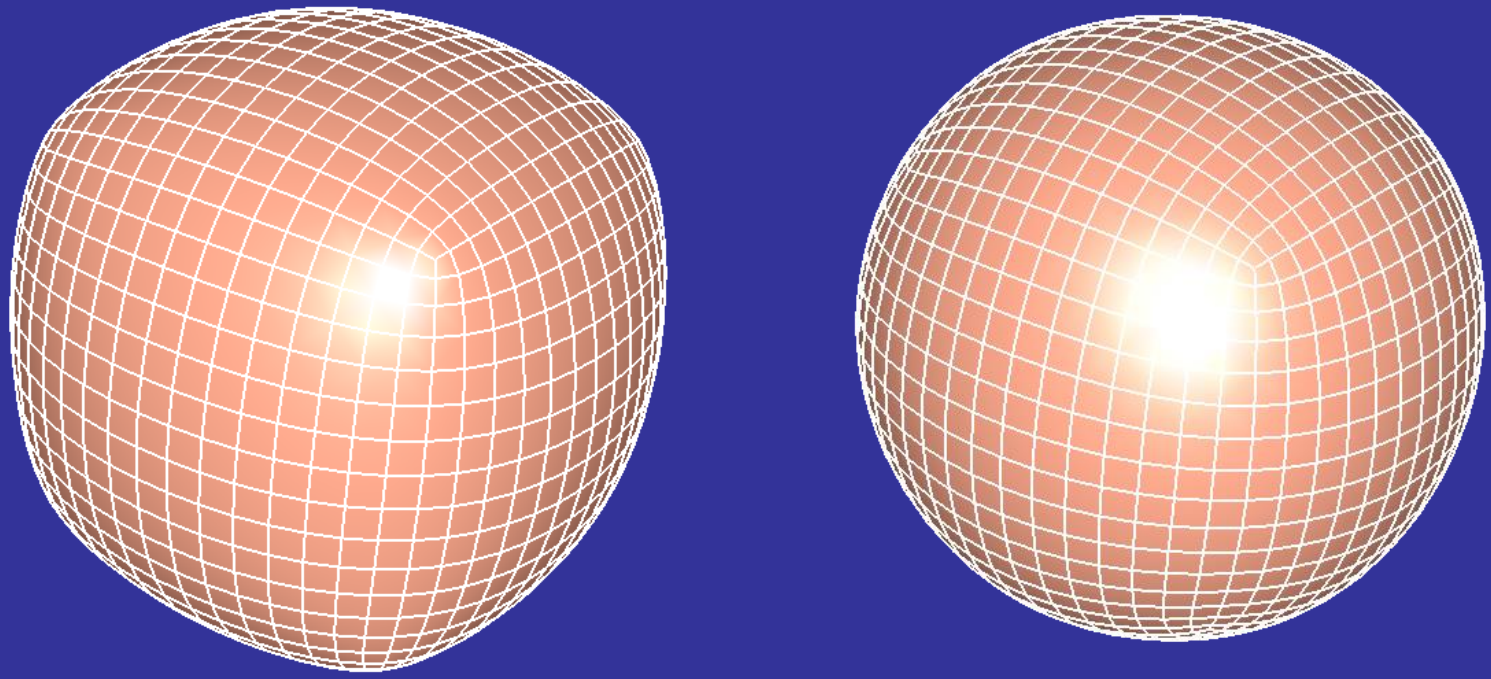
$$\alpha + N_e\,\alpha/2 + N_q\,\alpha/4 = 1$$

Some Examples



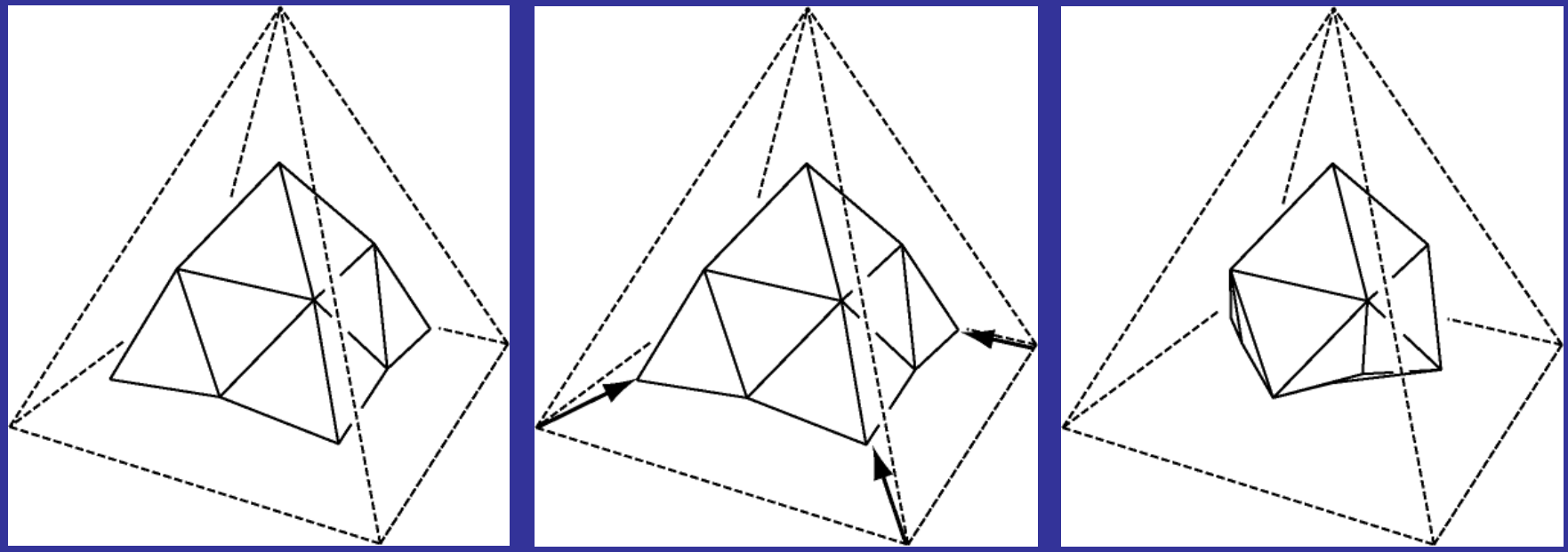
Surfaces “pinched” at corners

Vertex Correction



First used by Catmull-Clark

Vertex Correction

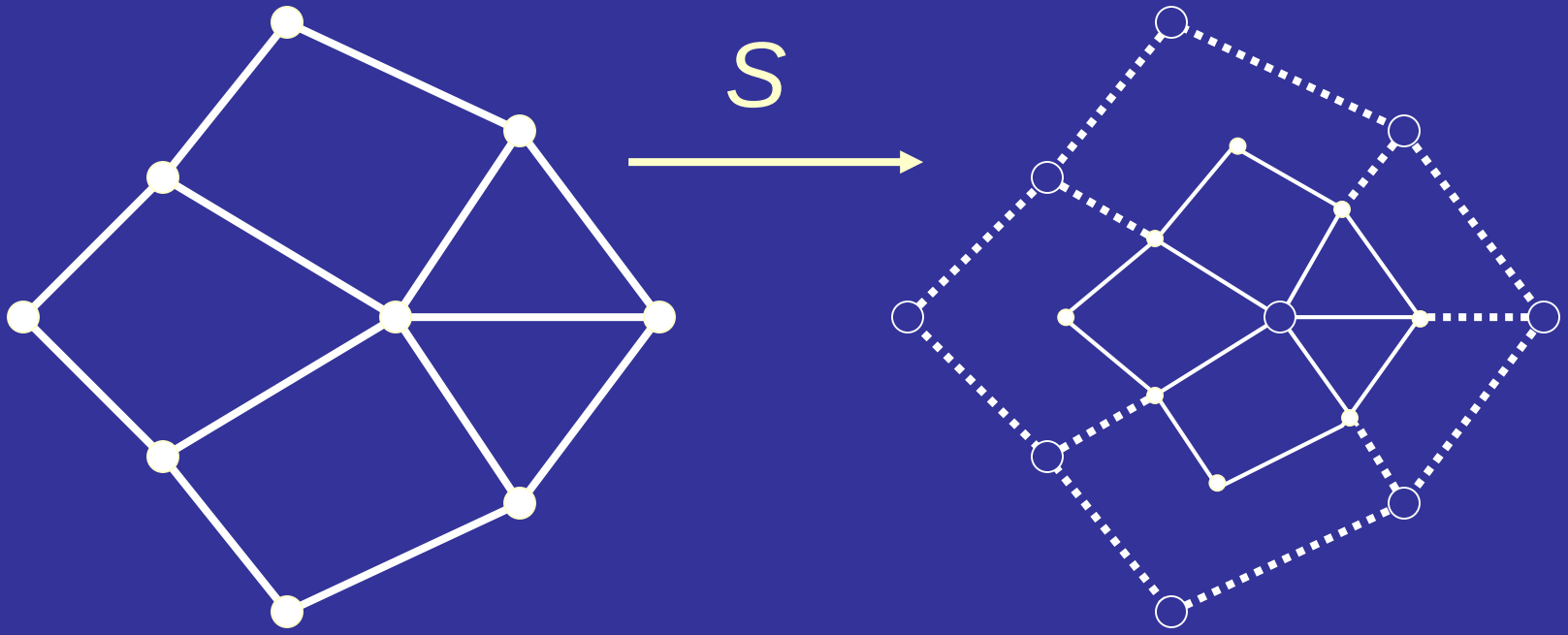


$$v^2 = v^1 + g\,(\,v^1 - v^0\,)$$

What is the optimal factor ?

(CC: $g = (4\text{-}N)/N$)

Eigen-structure



$$x^1 = S\,x^0$$

Eigen-values

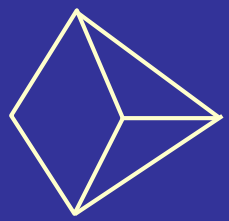
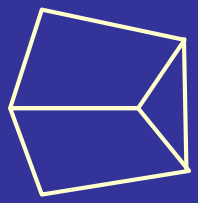
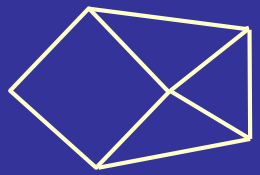
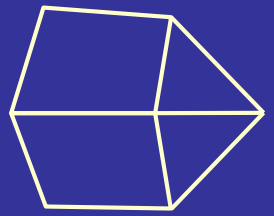
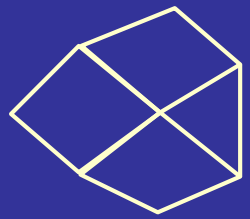
$$1 > \lambda \geqslant \lambda_2 > \mu \geqslant \lambda_4 \geqslant \lambda_5 > \ldots$$

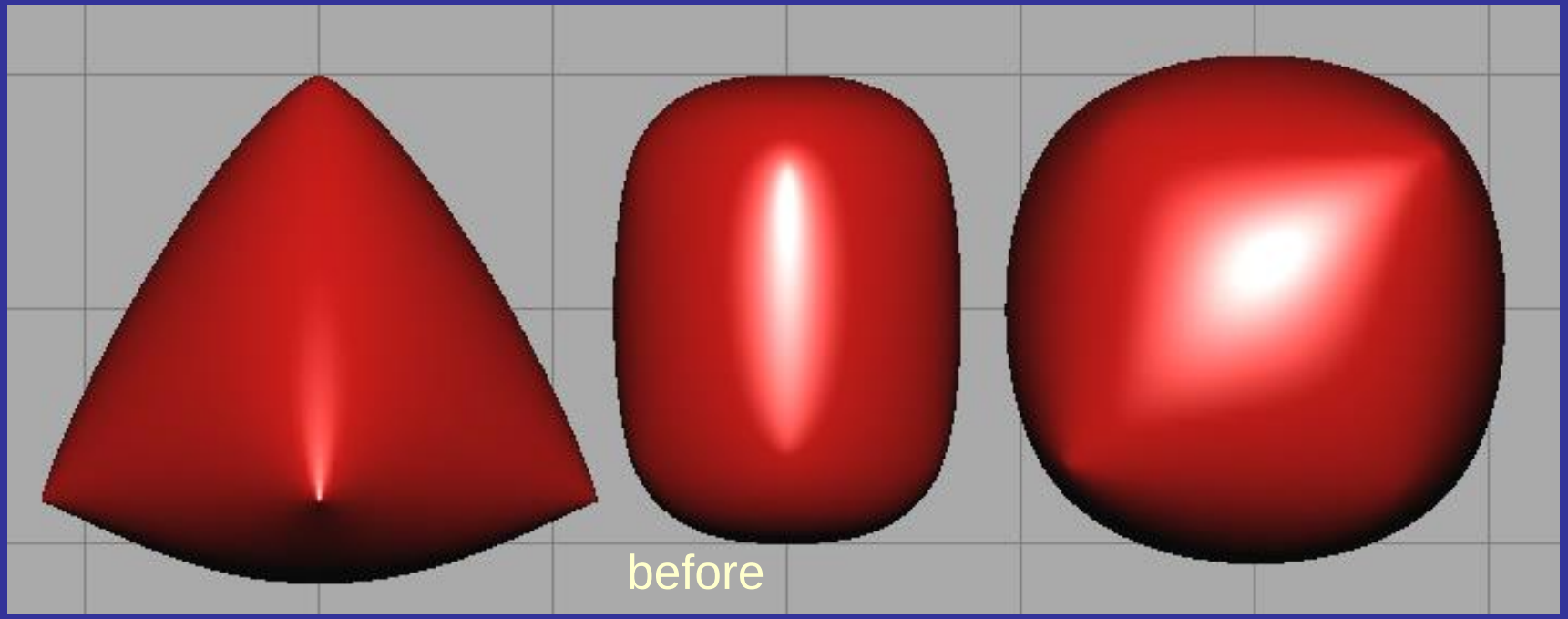
Bounded curvature: $\mu = \lambda^2$

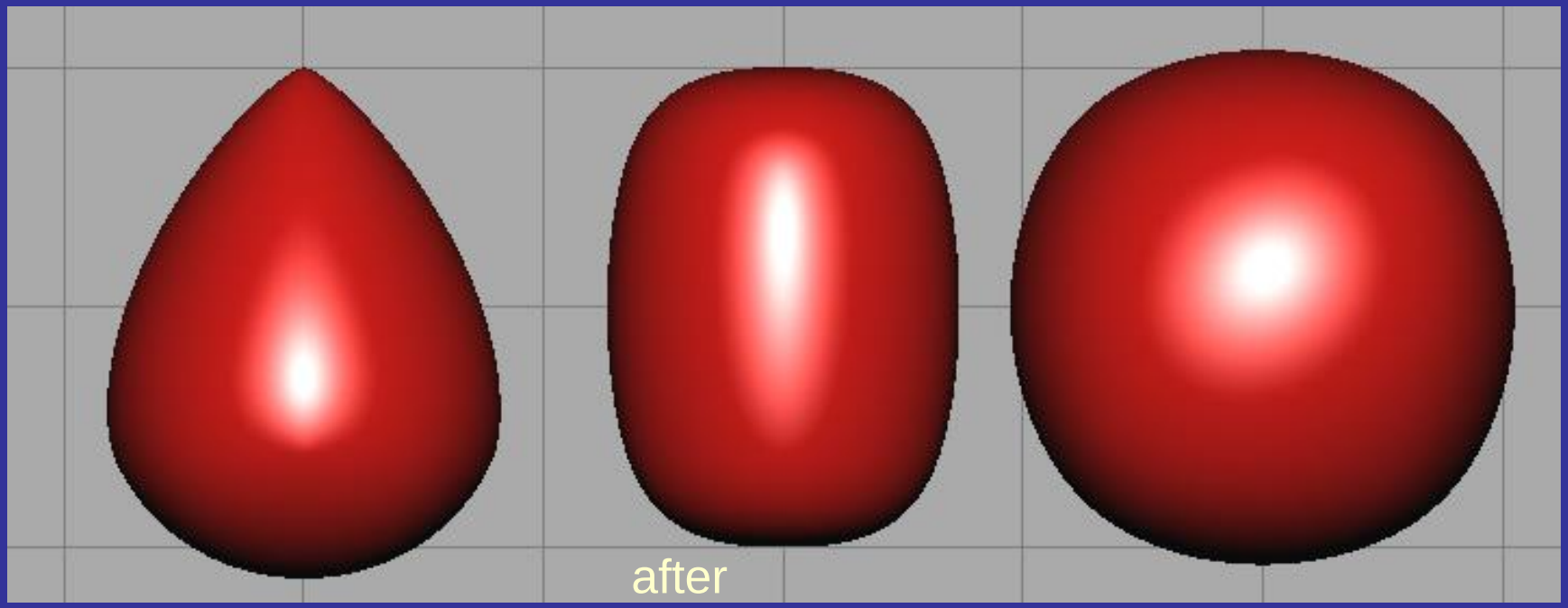
Define ratio: $\rho = \mu / \lambda^2$

We want: $\rho = 1$

Back to Vertex Correction

					
g	0.80597	0.61539	0.34792	0.21380	0.10550
ρ	1.227	1.242	1.000	1.000	1.000


before


after

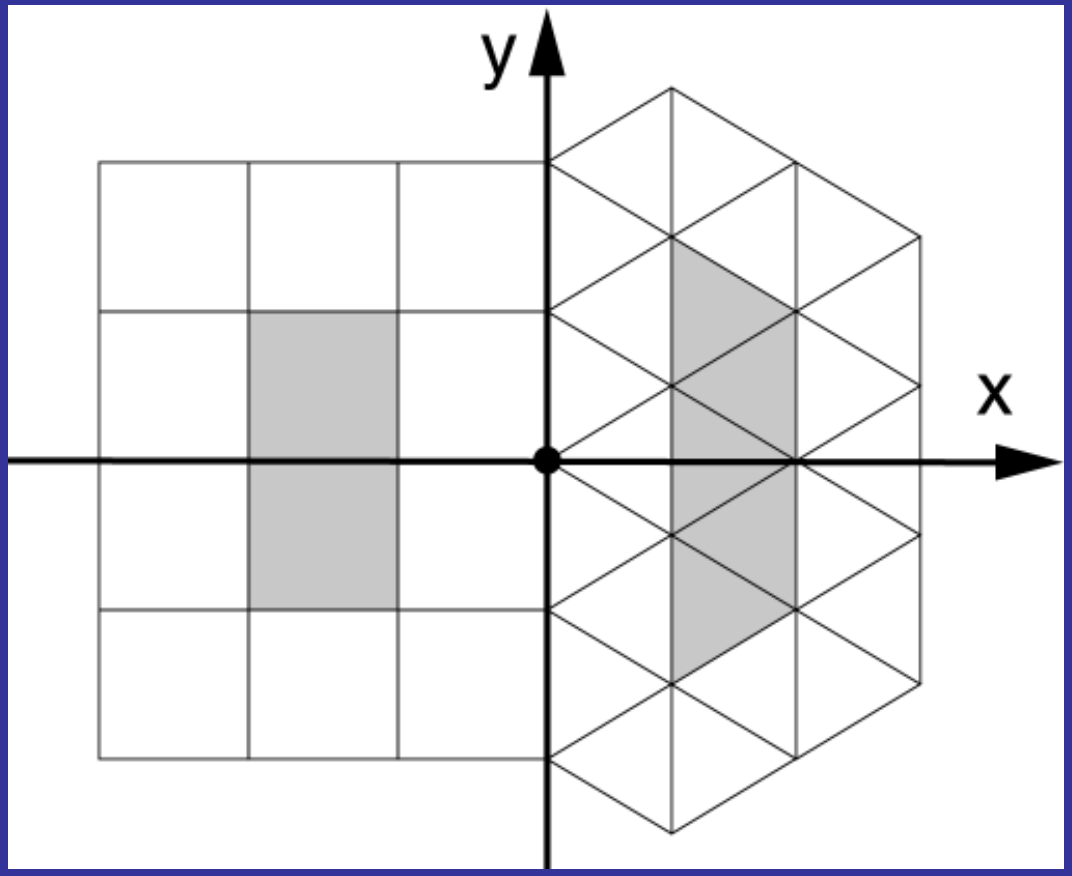
Smoothness

C^2 everywhere except at:

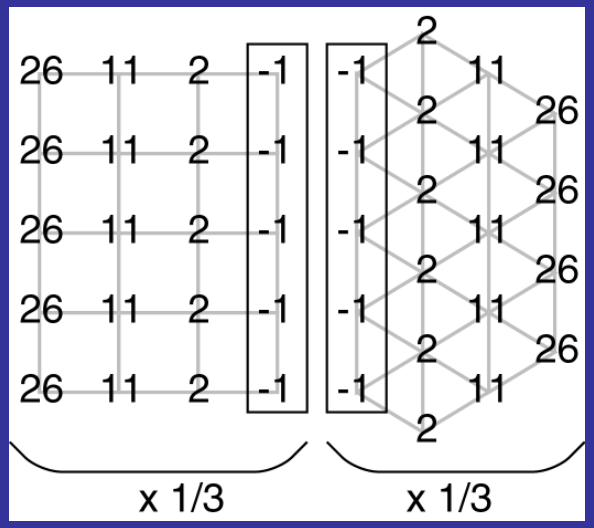
Quad/triangle boundary

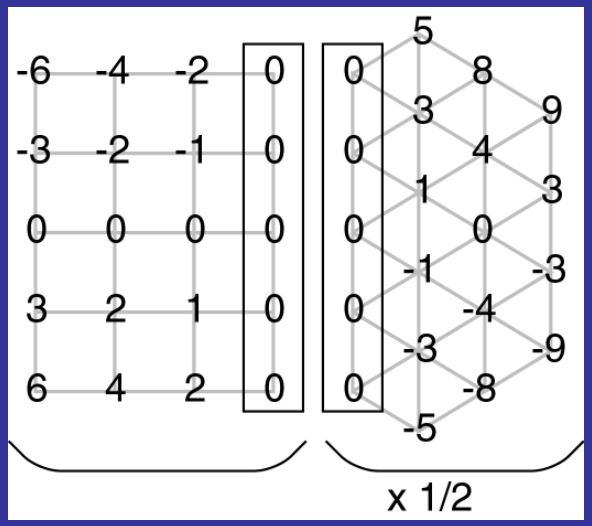
Extraordinary vertices

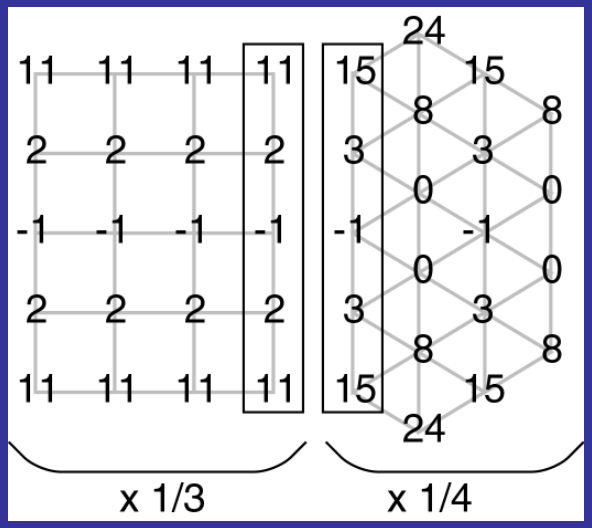
Regular Case



Regular Case not C^2



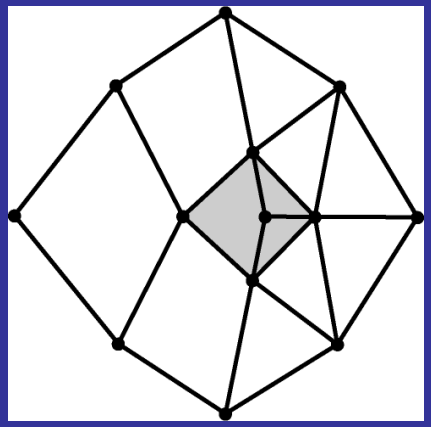
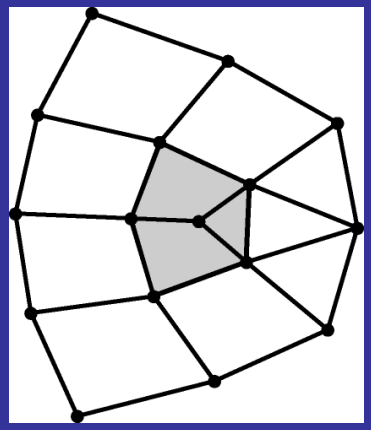
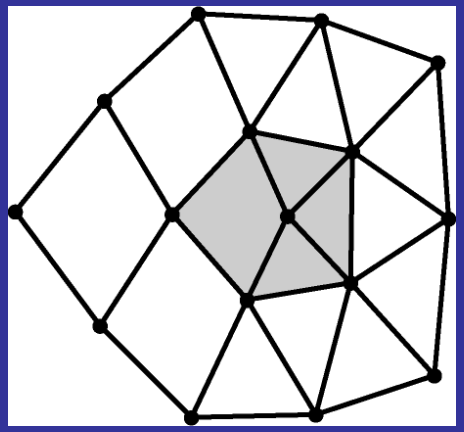
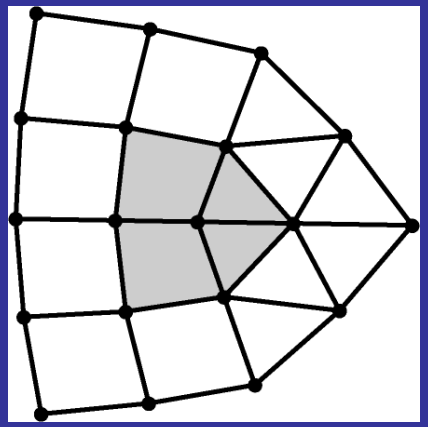
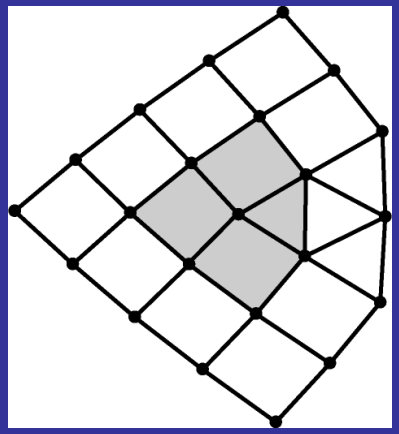
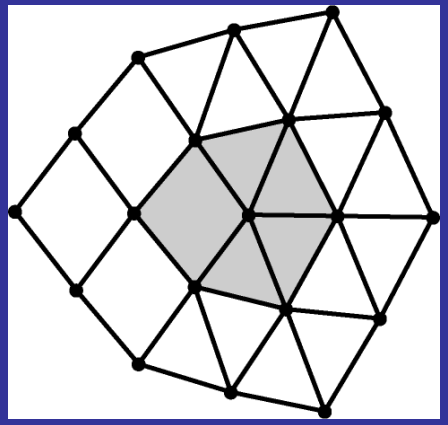
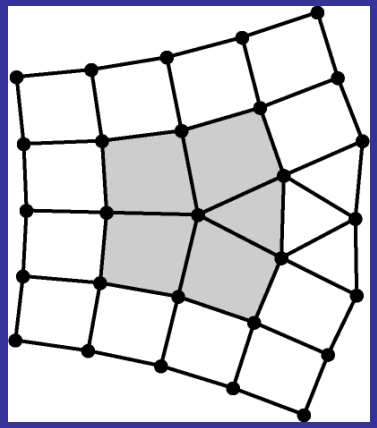
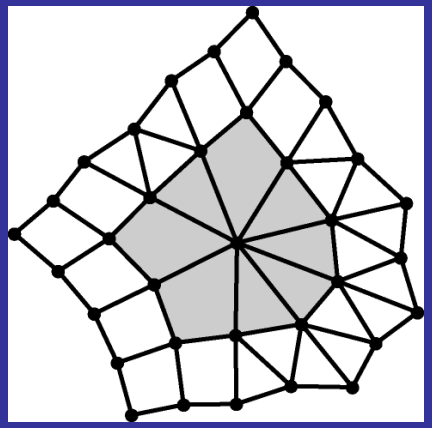




x^2 xy y^2

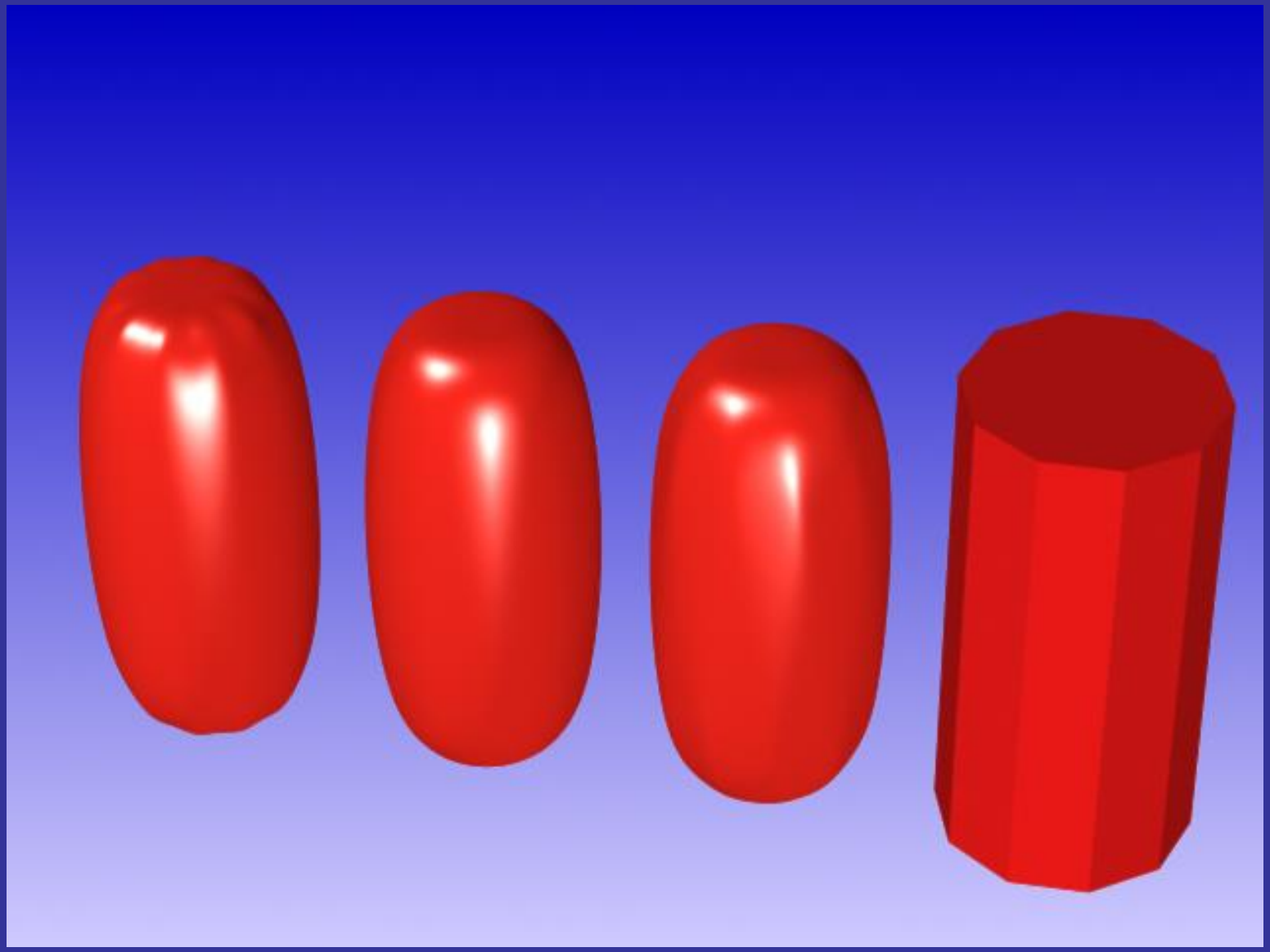
Eigen-values: 1, ½, ½, ¼, ¼, ¼, …

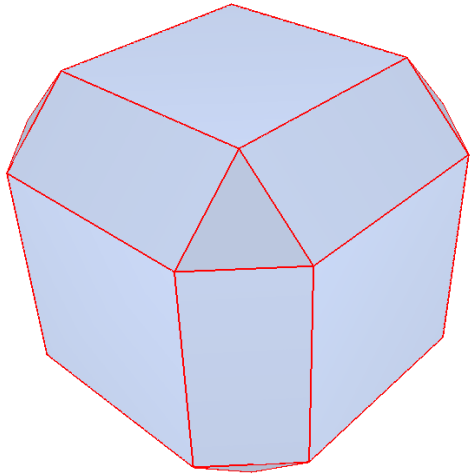
Irregular Case

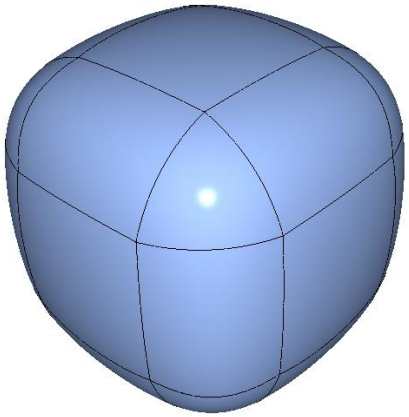









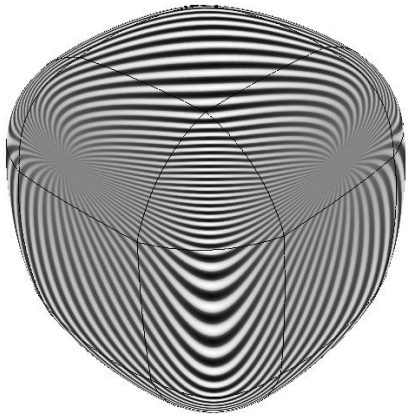
Examples

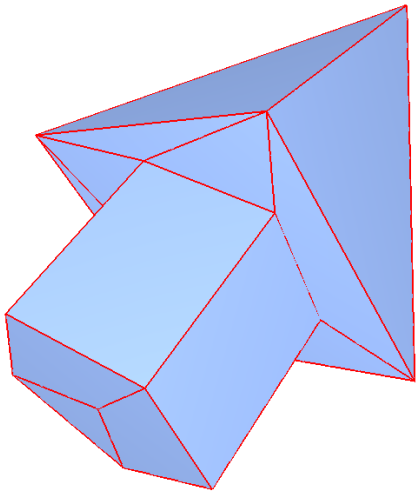
MAYA shape plugin

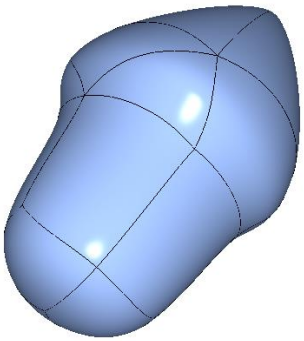


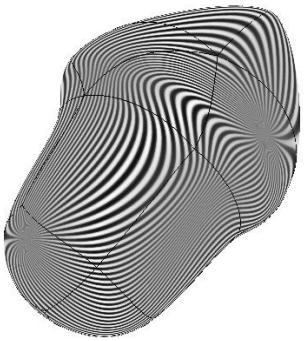


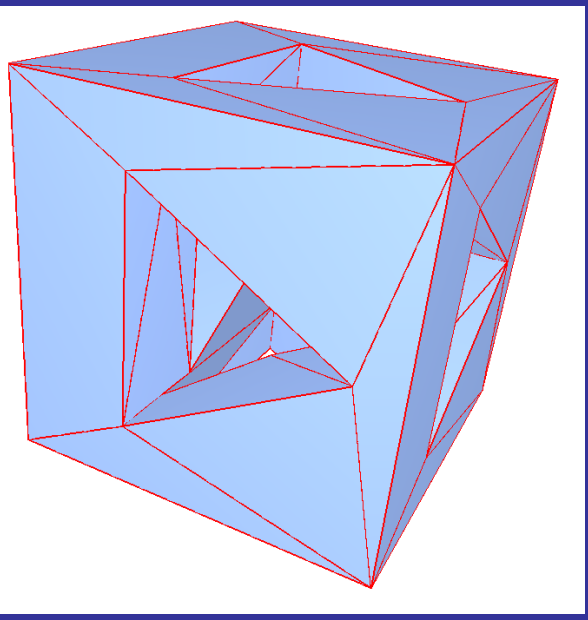
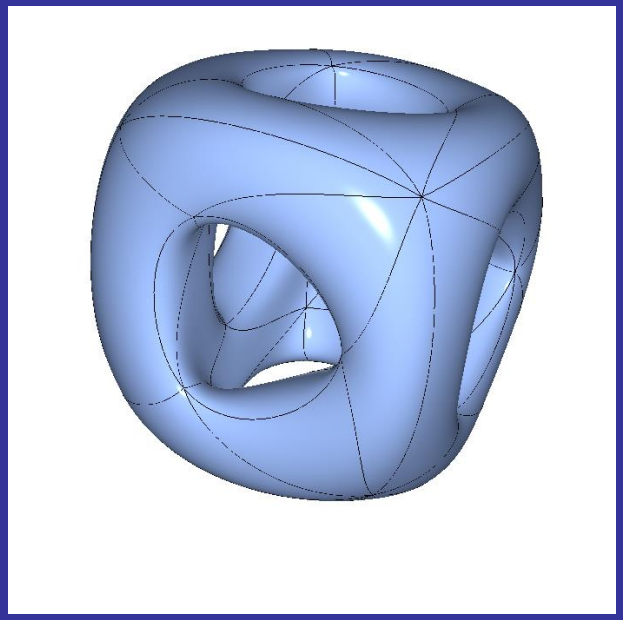
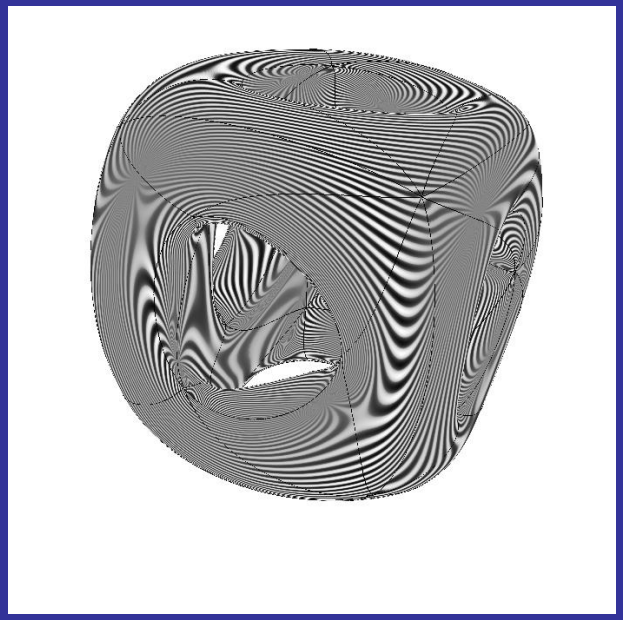
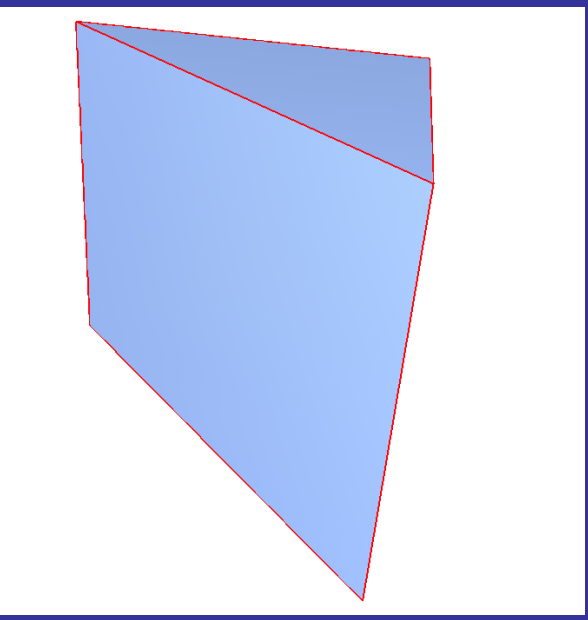
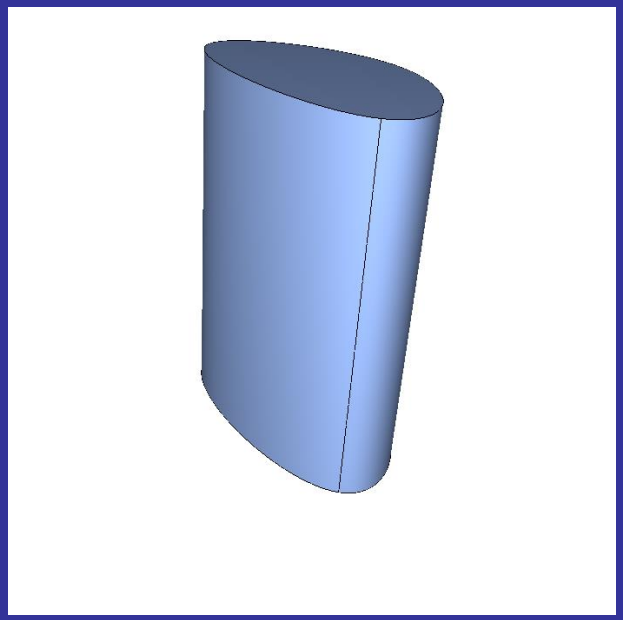
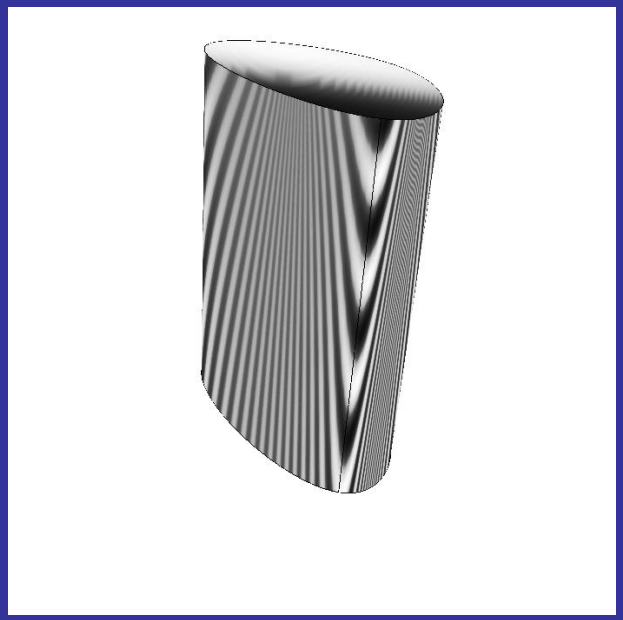










Future Work

Exact Evaluation

Formal C^1 proof

Better rules for “bad” meshes