

# Fluid Control Using the Adjoint Method

Antoine McNamara

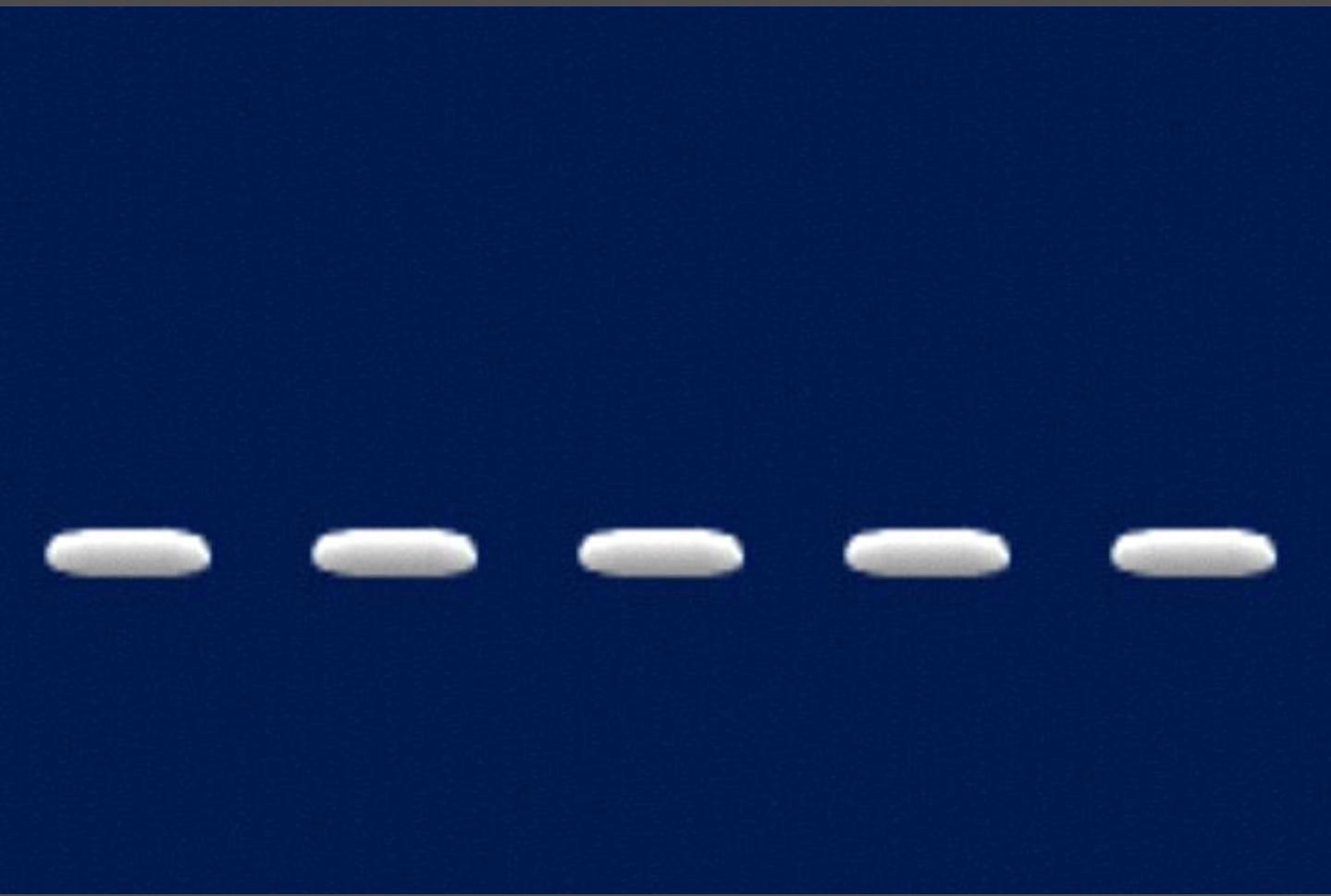
Adrien Treuille

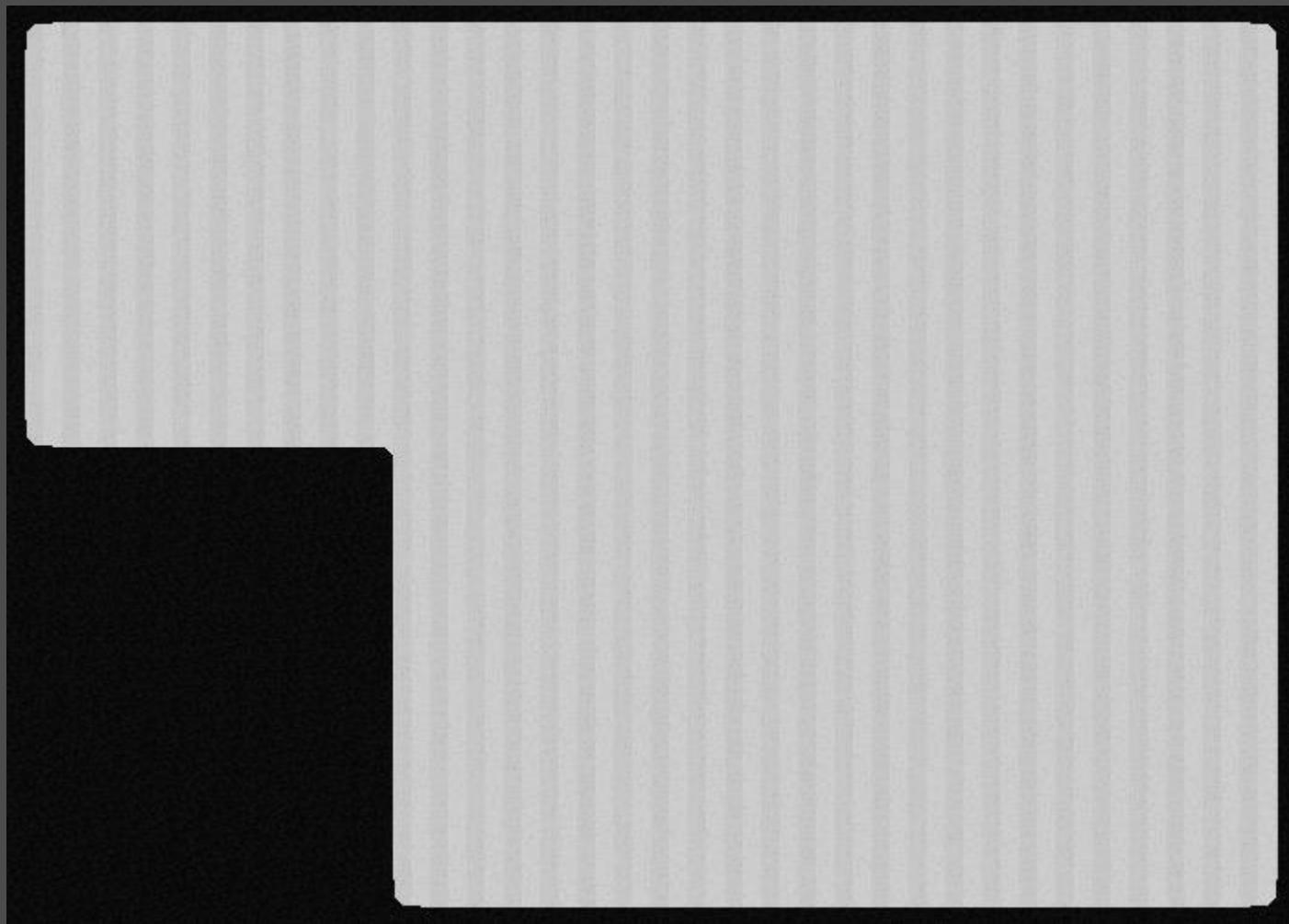
Zoran Popović

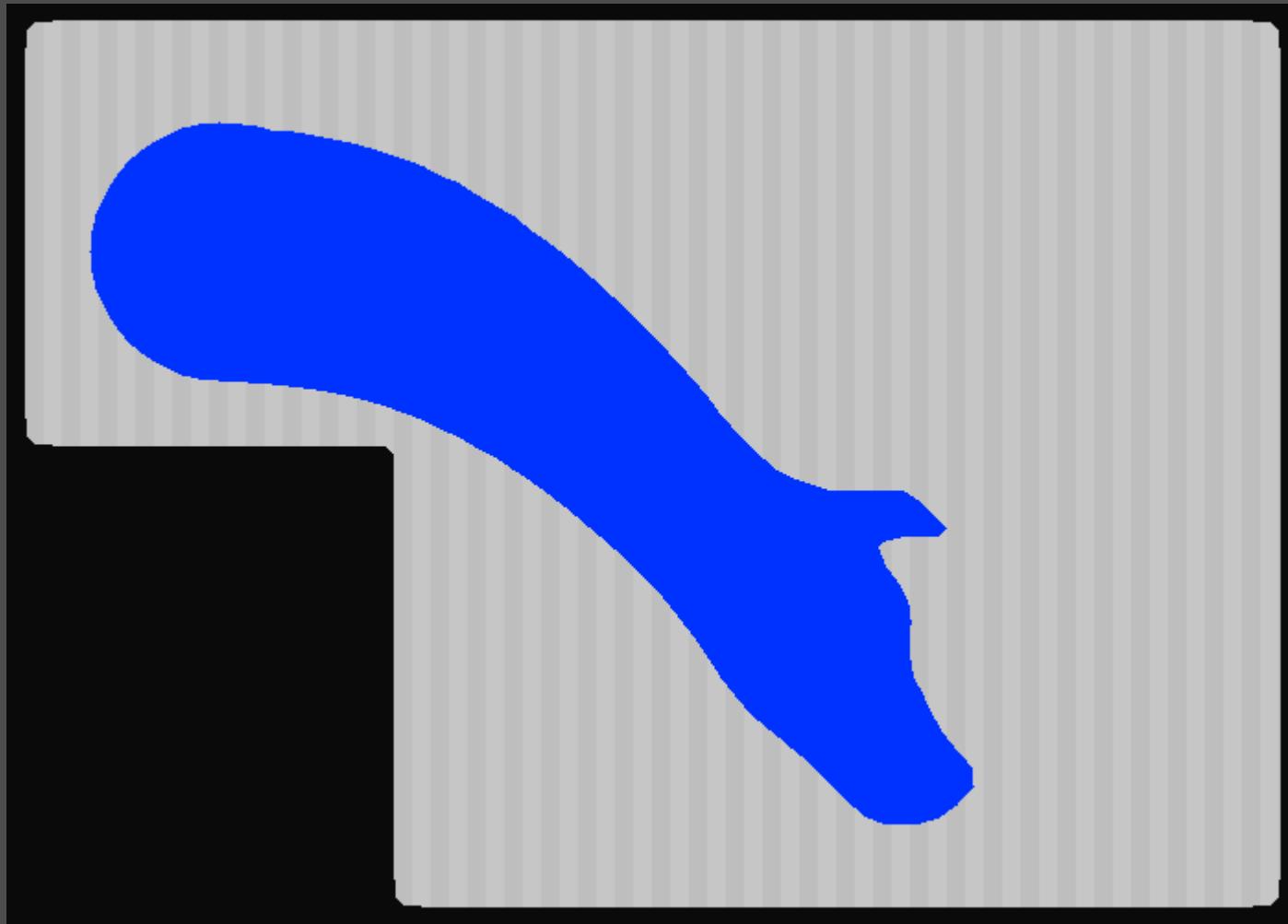
University of Washington

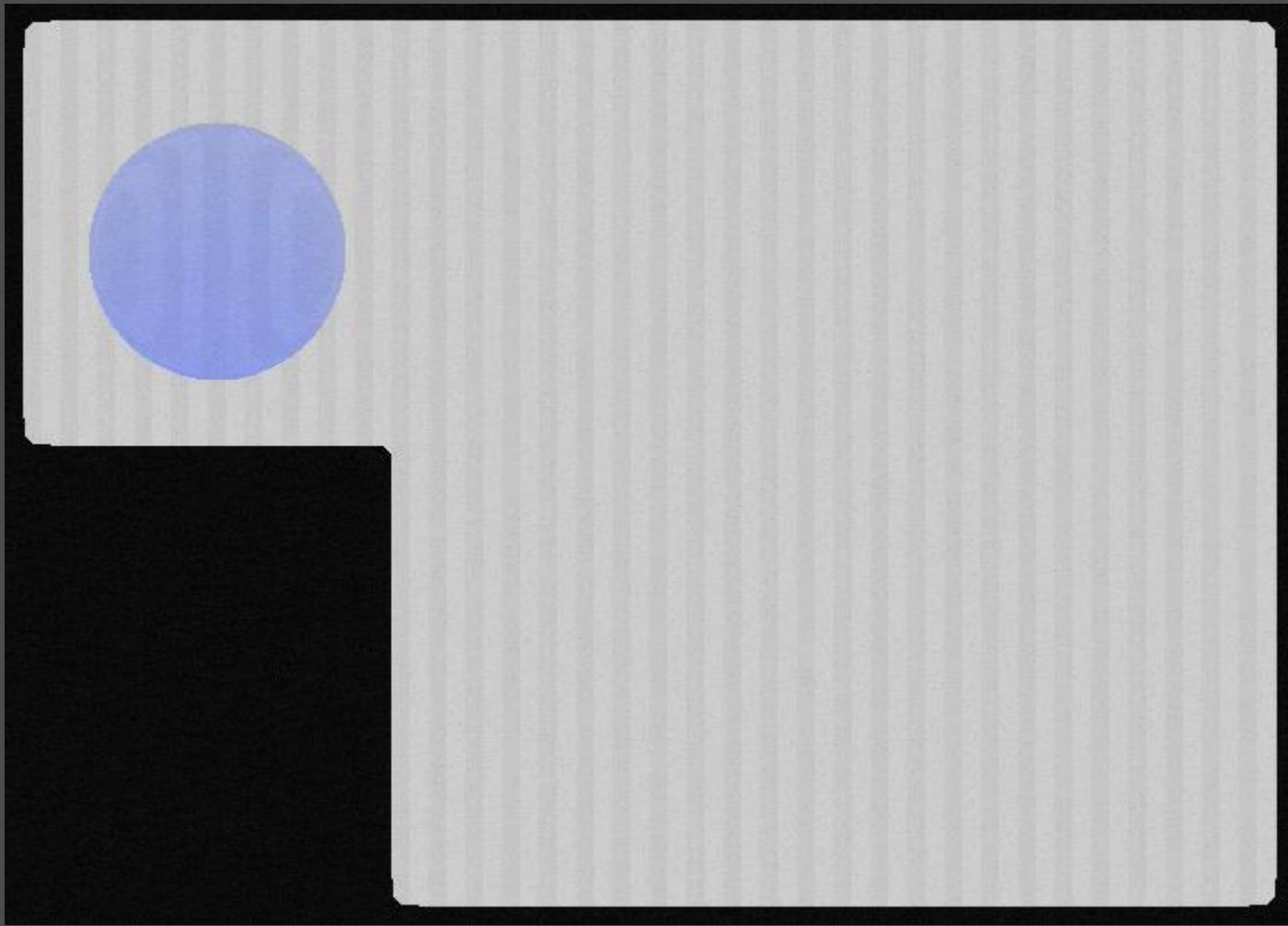
Jos Stam

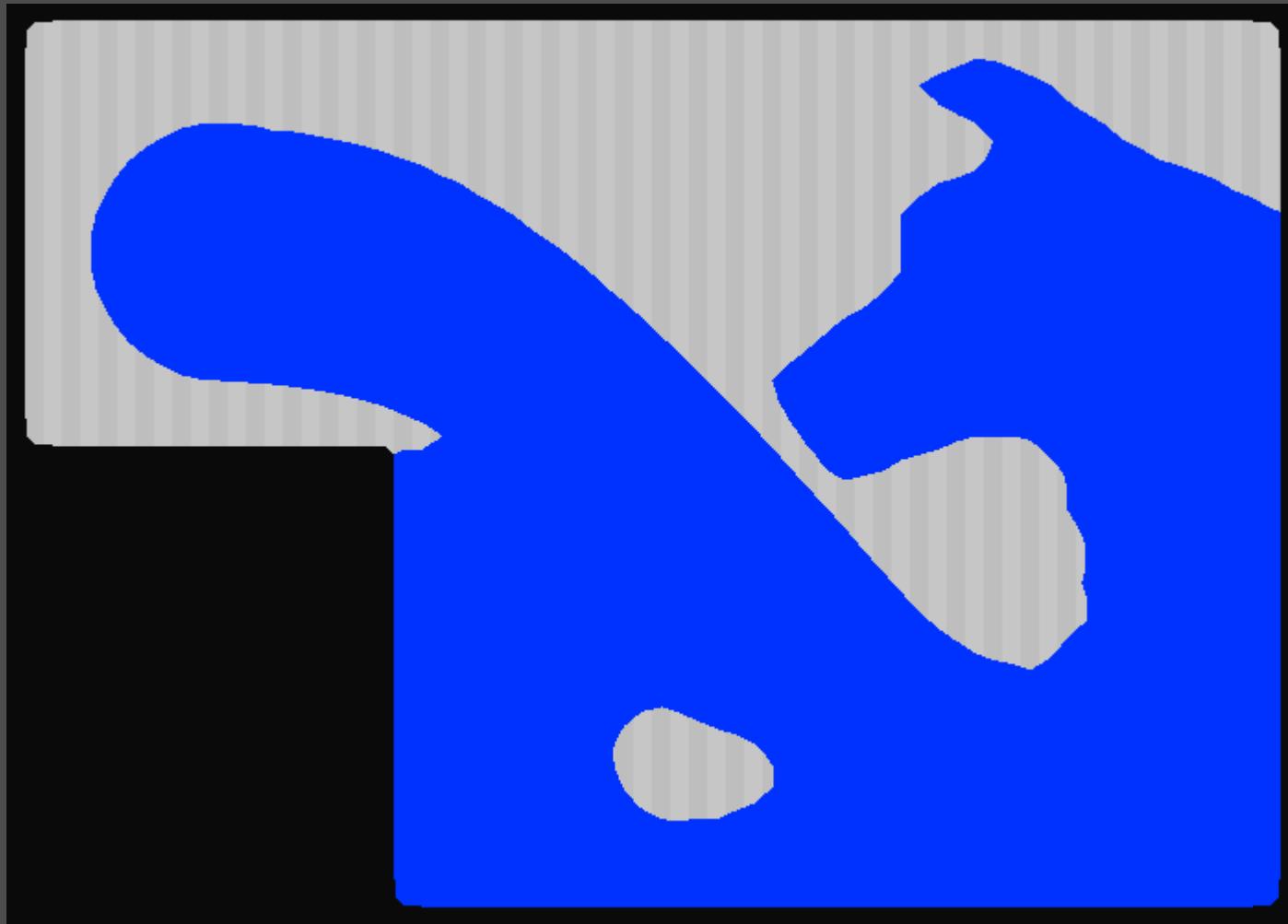
Alias Systems

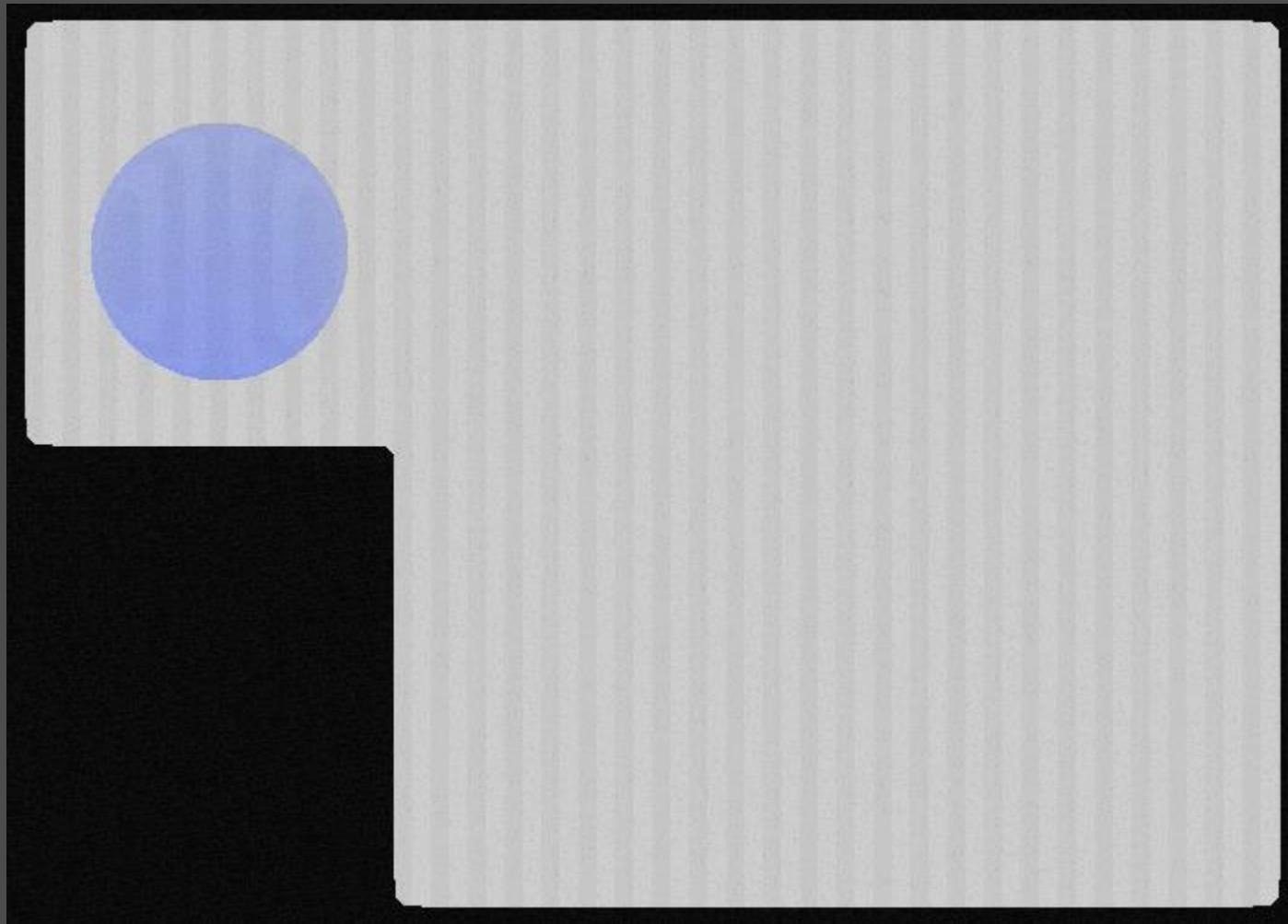












# Related Work

# Fluids Simulation in Graphics

- Smoke
  - Foster and Metaxes [1997]
  - Stam [1999]
  - Fedkiw et al. [2001]
- Water
  - Foster and Fedkiw [2001]
  - Enright et al. [2002]
- New
  - Carlson et al. [2004]
  - Losasso et al. [2004]
  - Goktekin et al. [2004]



# Simulation Control in Graphics

- Barzel et al. [1996]
- Chenney and Forsyth [2000]
- J. Popović et al. [2000]
- Treuille et al. [2003]
- Fattal and Lischinski [2004]

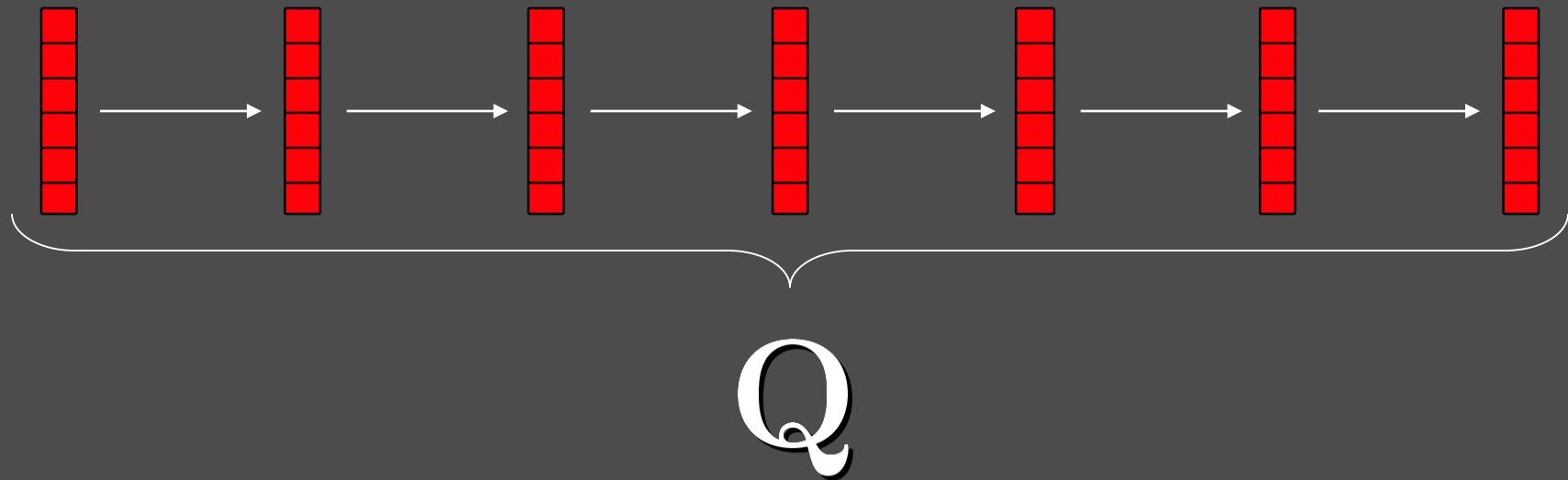
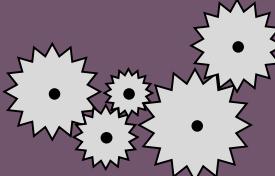


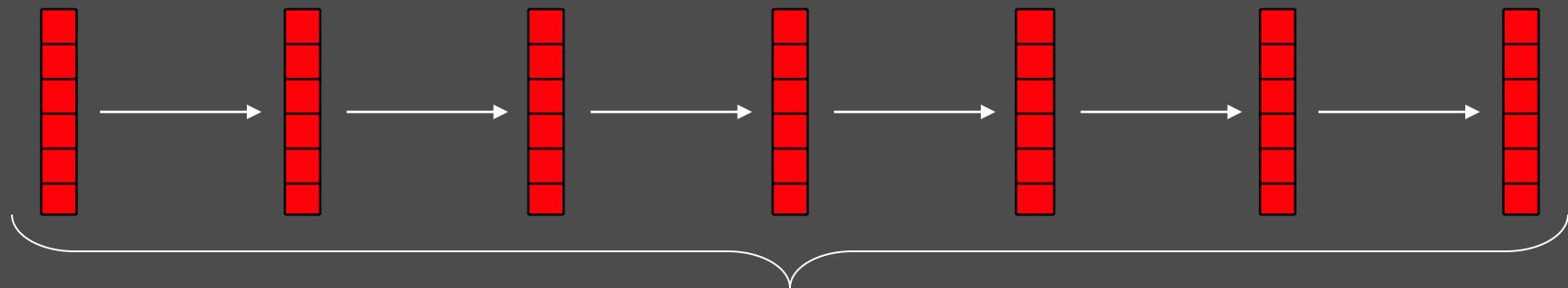
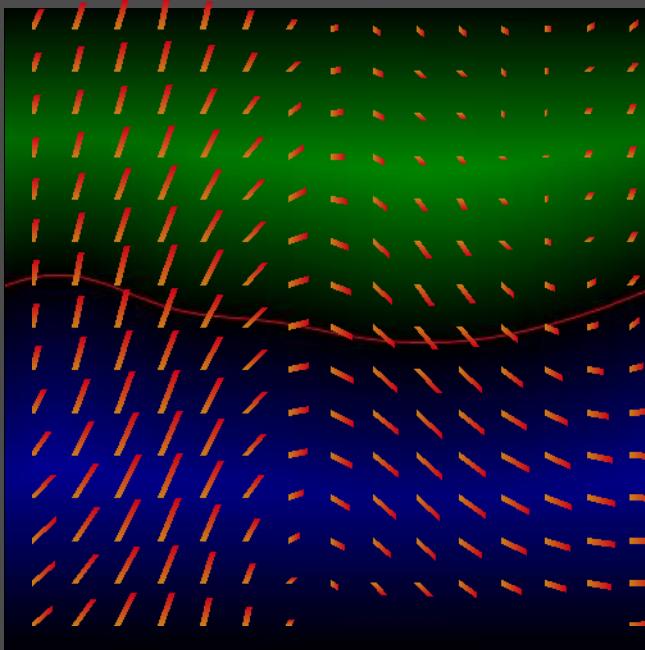
# The Adjoint Method

- Pironneau [1974]
- Jameson [1988]
- Giles and Pierce [2000]
- Bewley [2001]

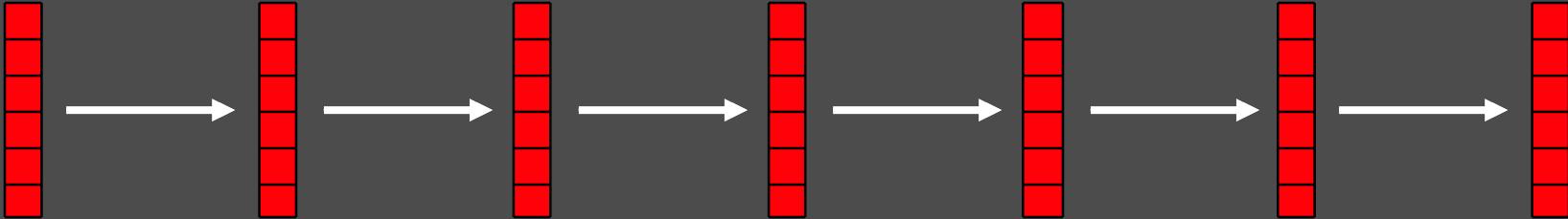
# Overview

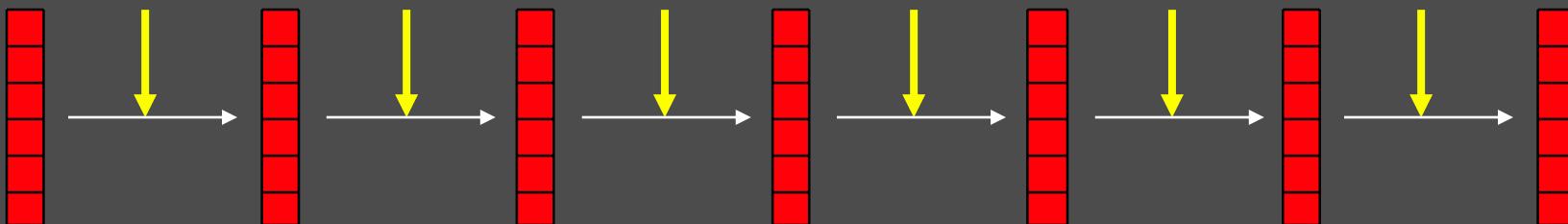
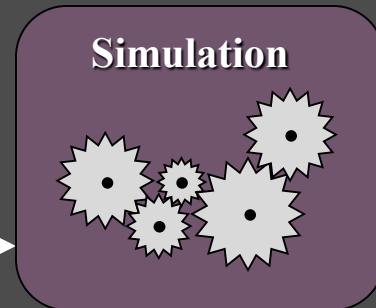
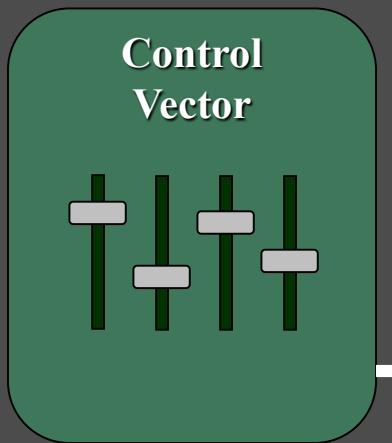
Simulation

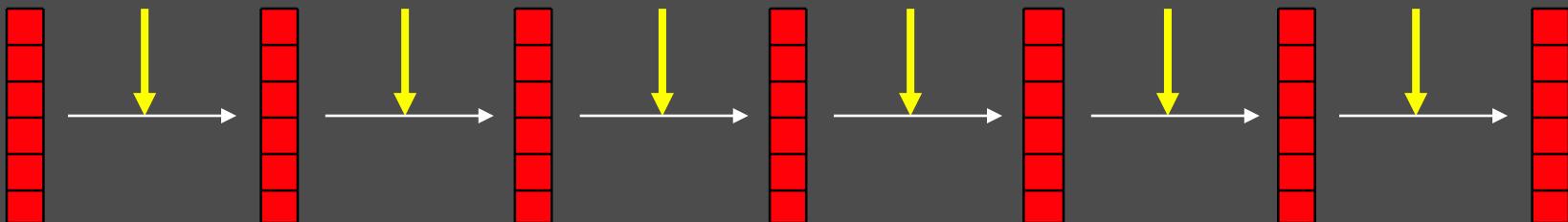
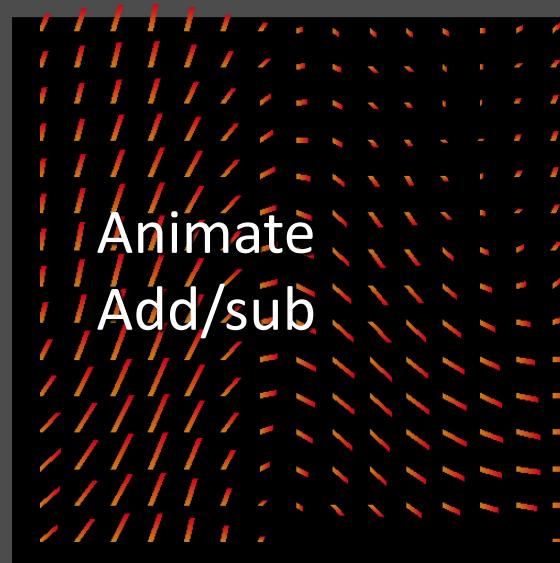


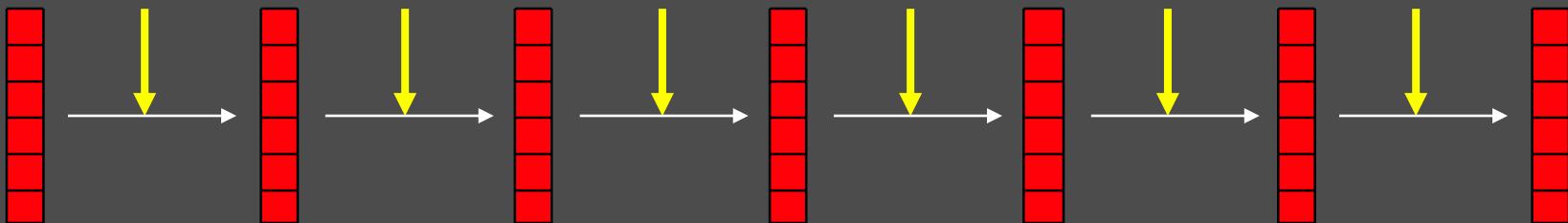
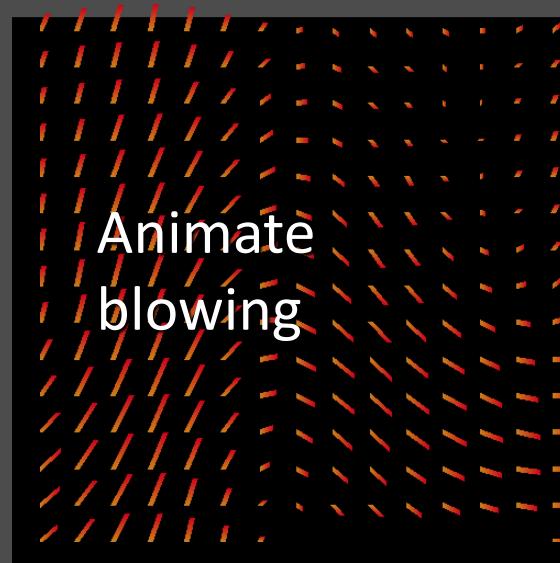


Q

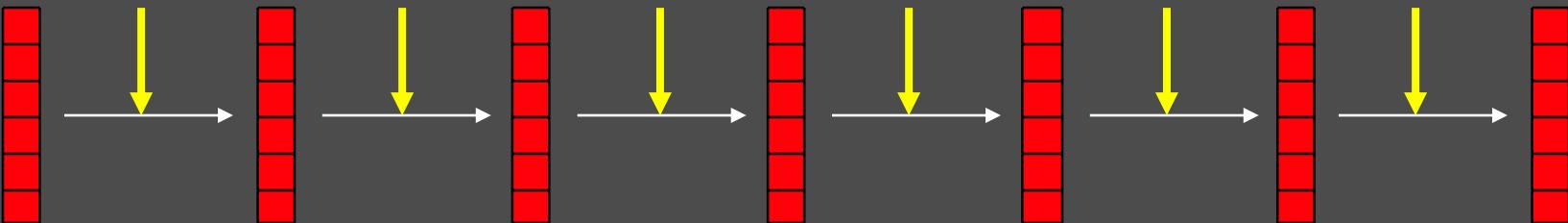


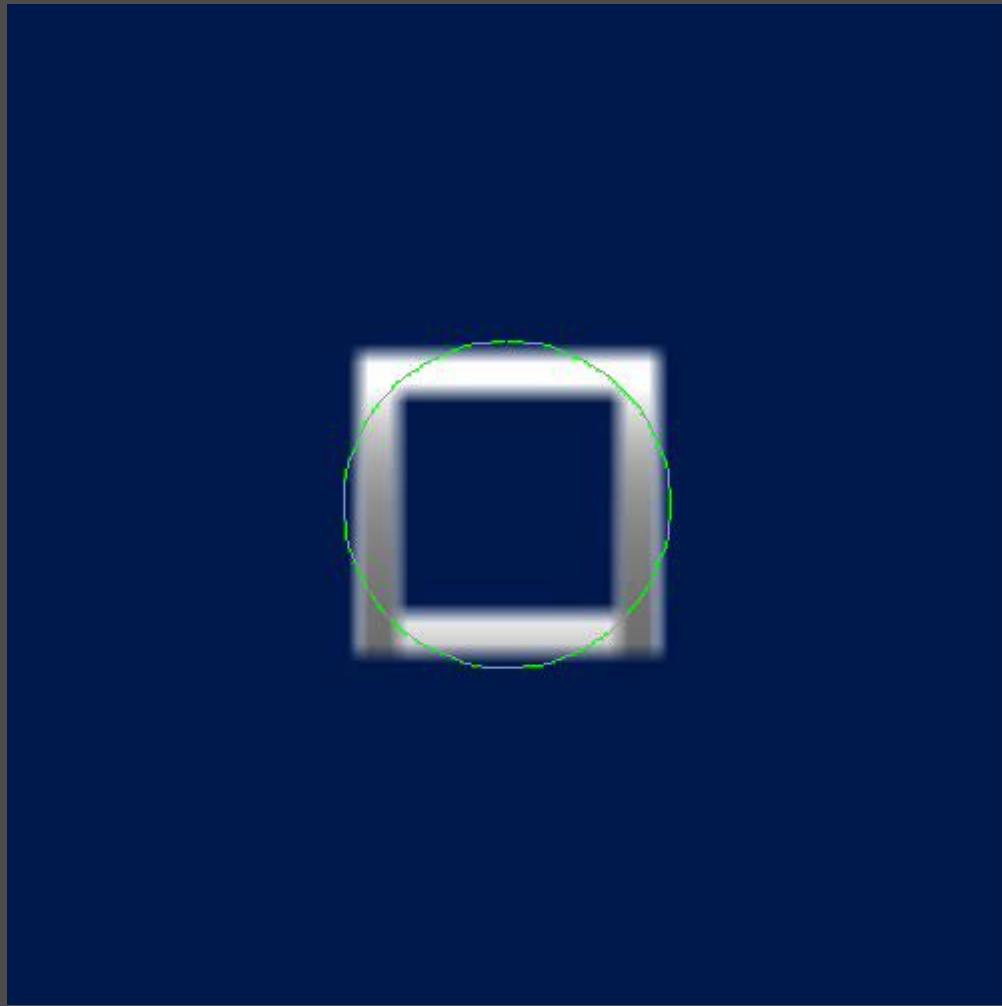


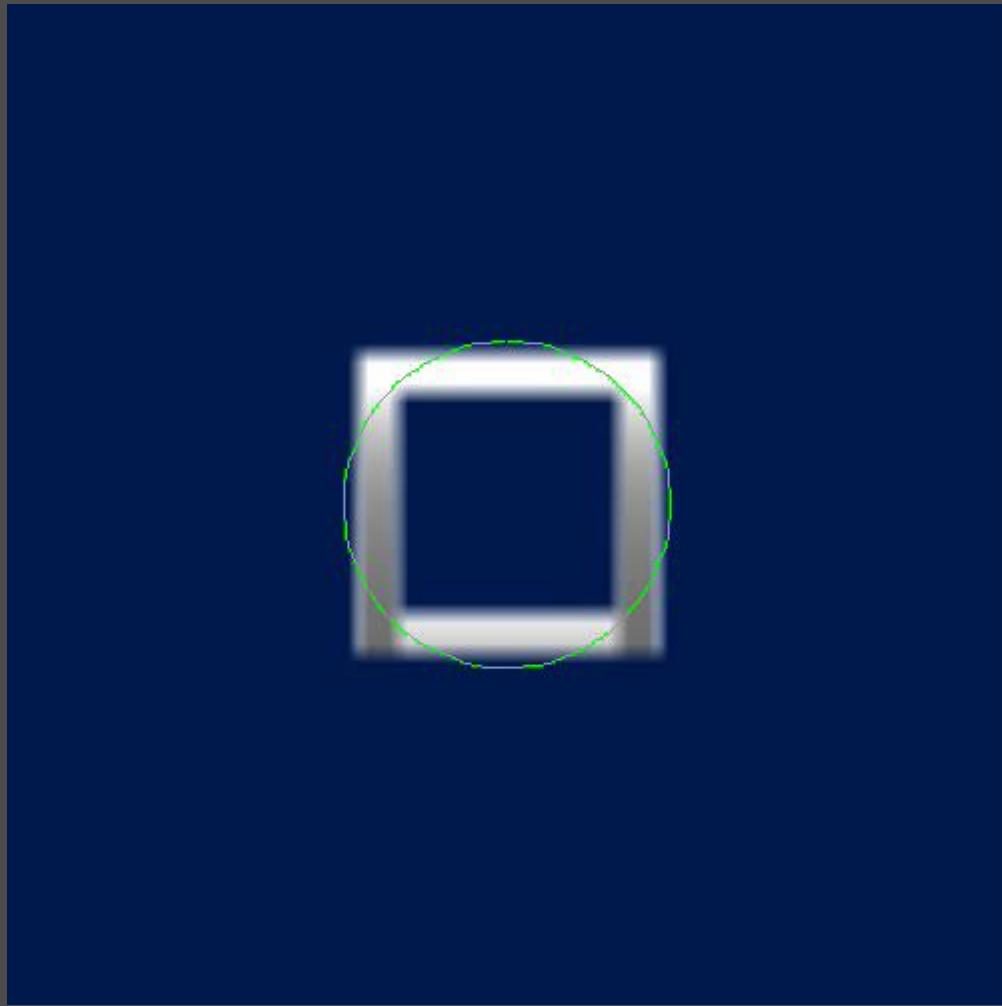


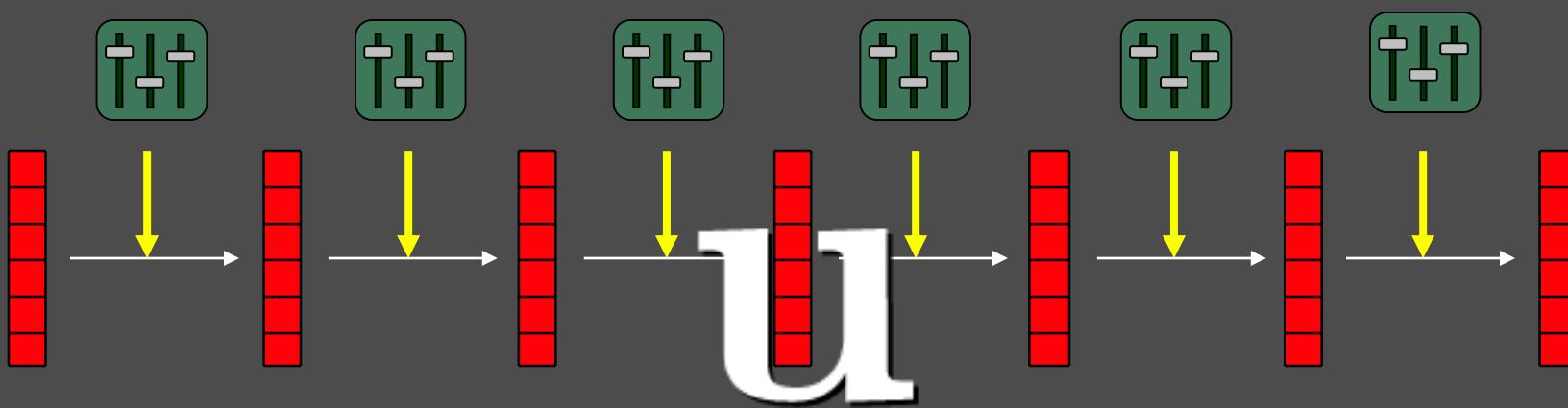
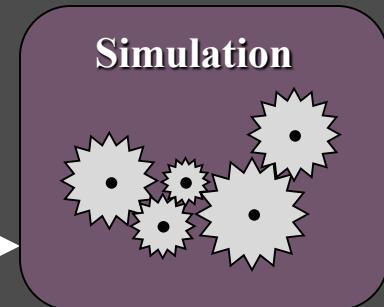
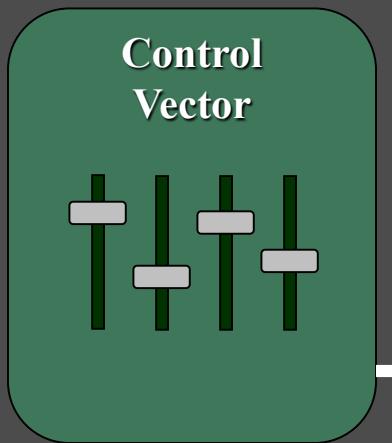


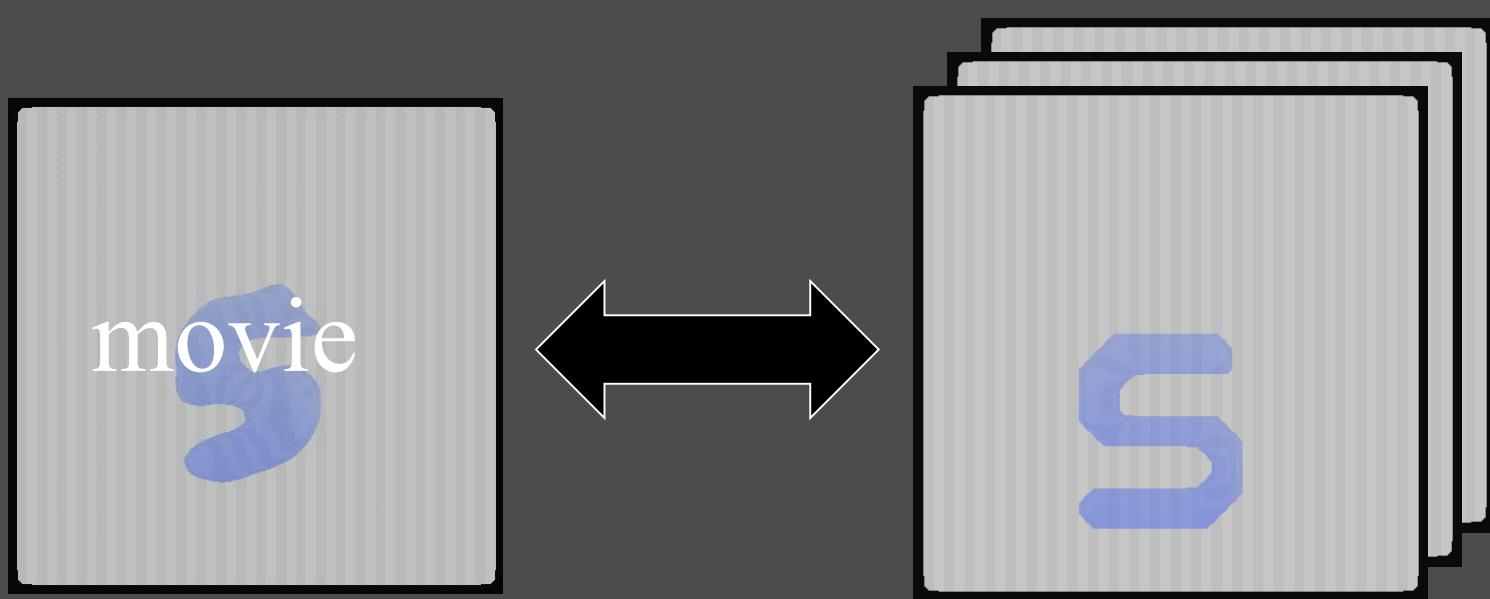
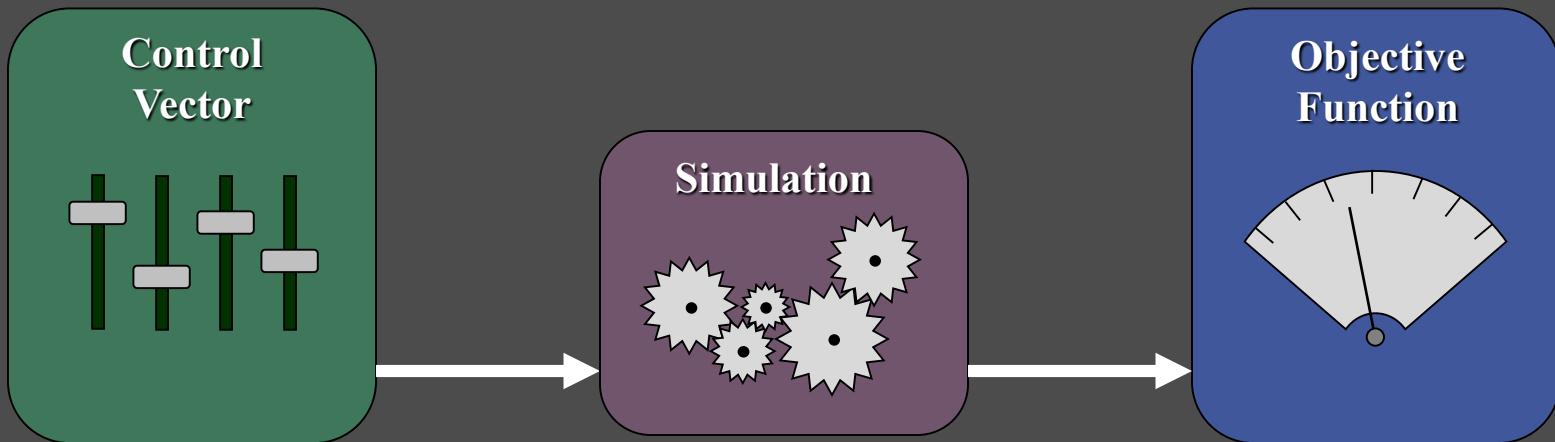
Animate  
add/sub



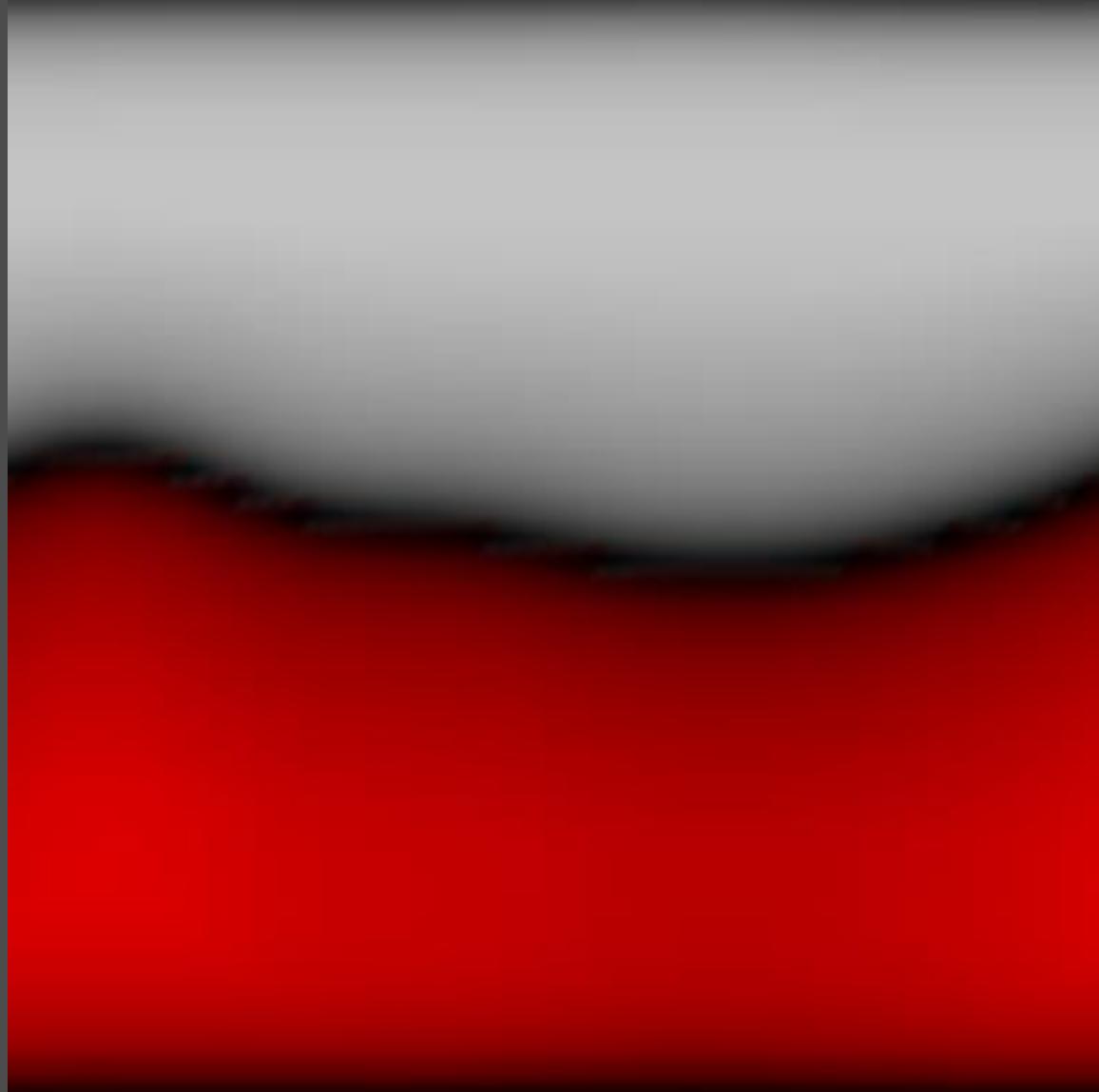




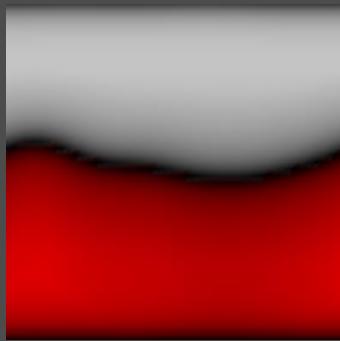




$$|c - c|^2$$



|

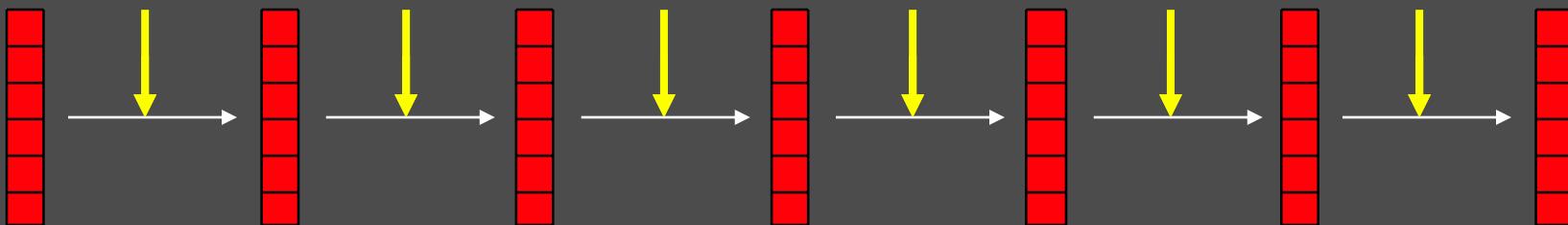
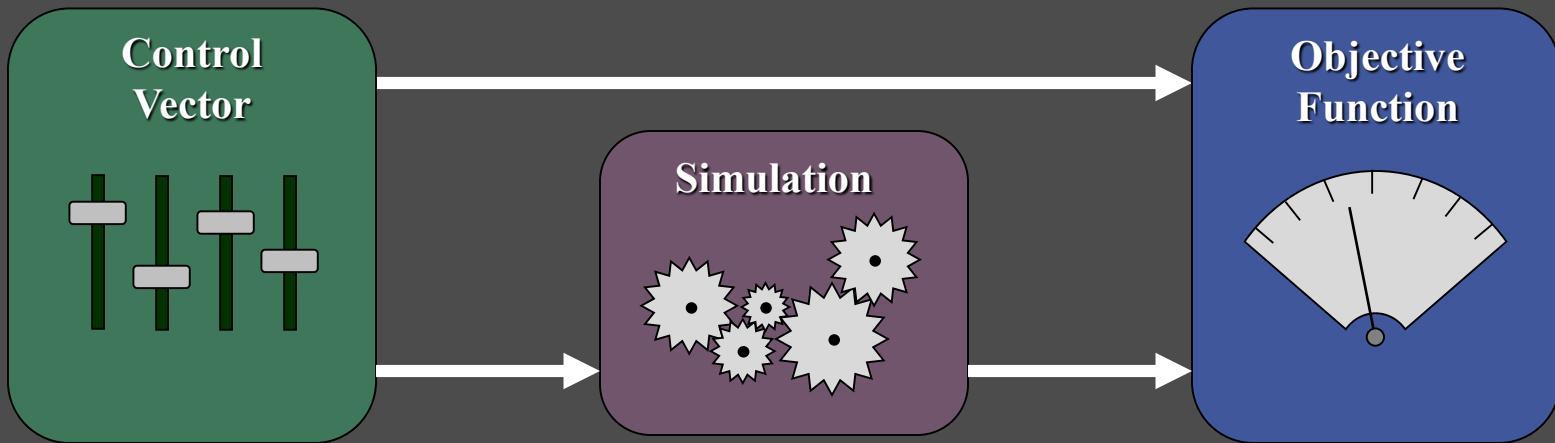


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2

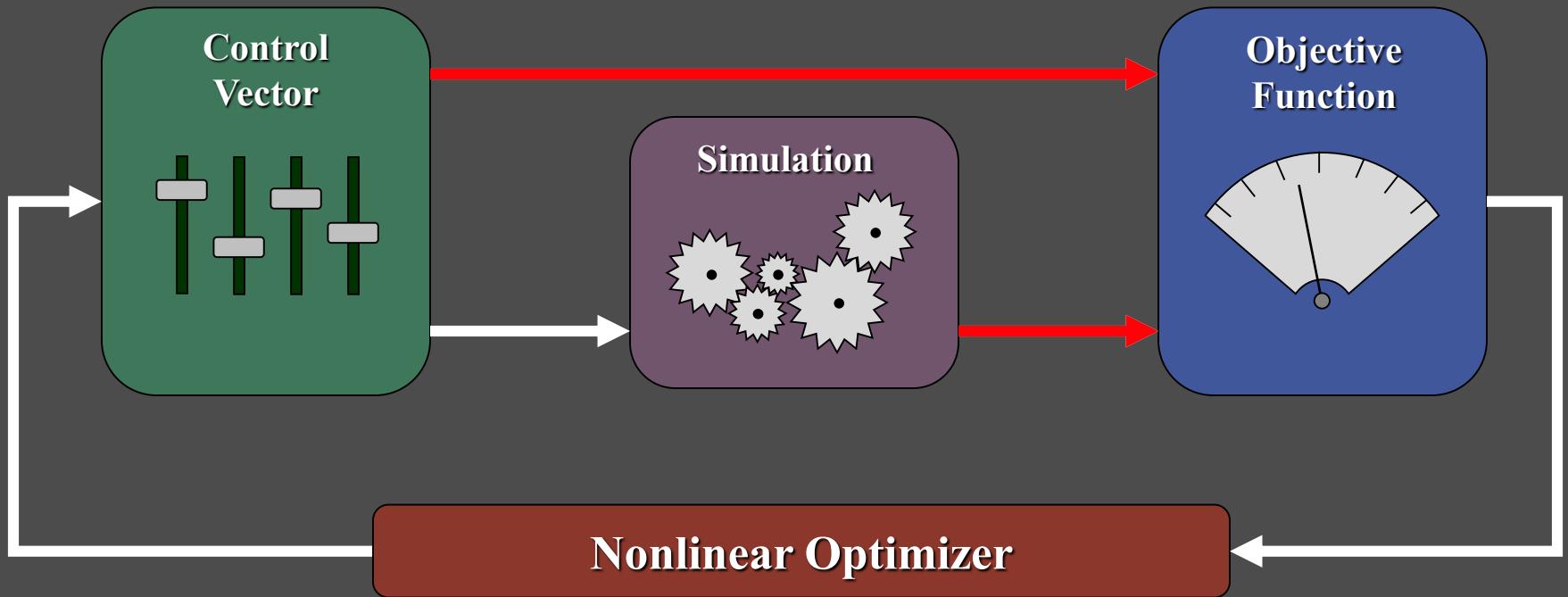




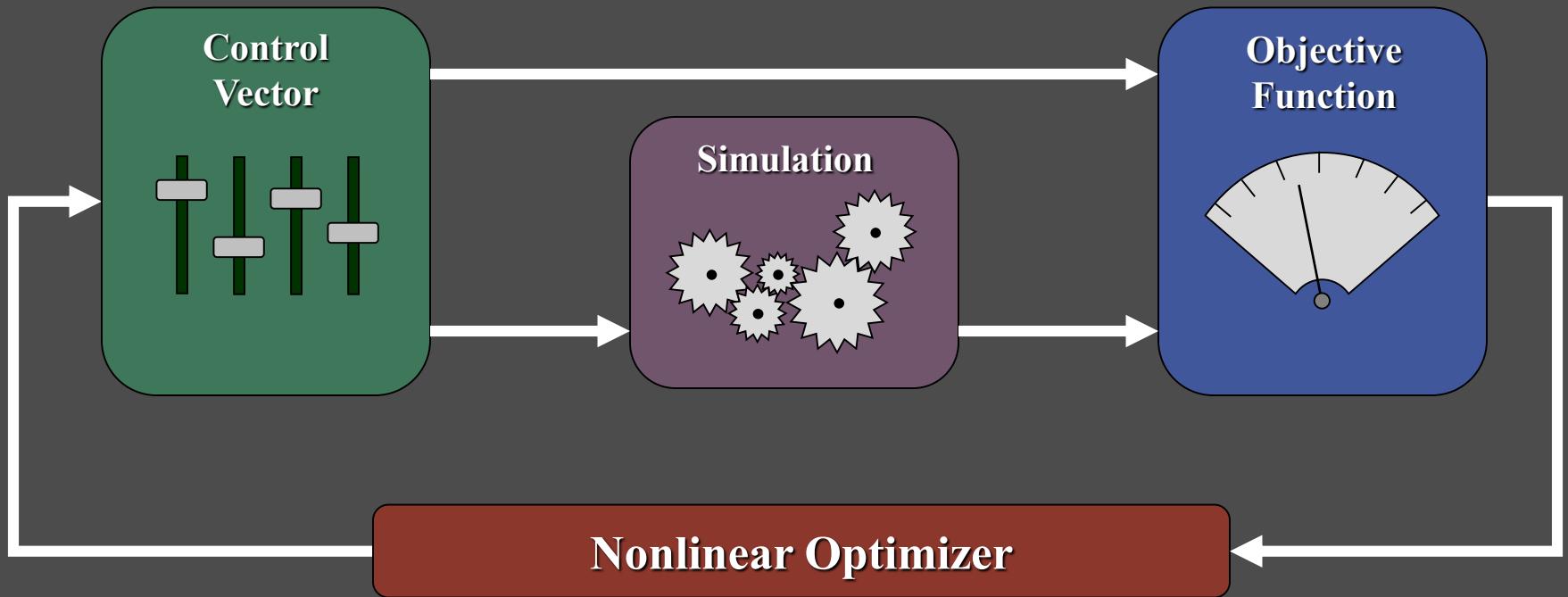








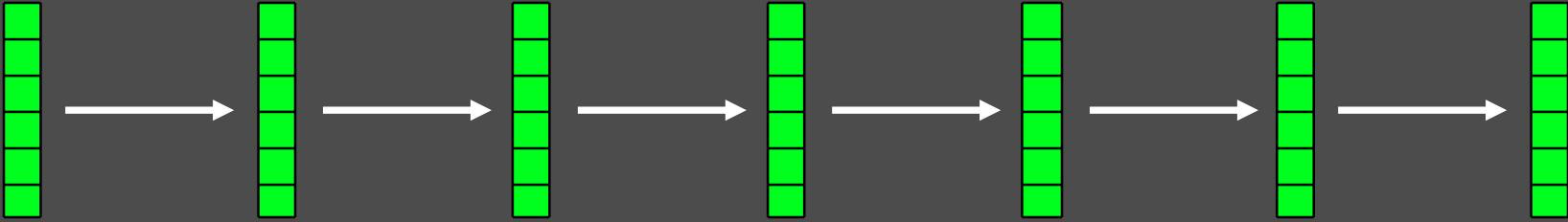
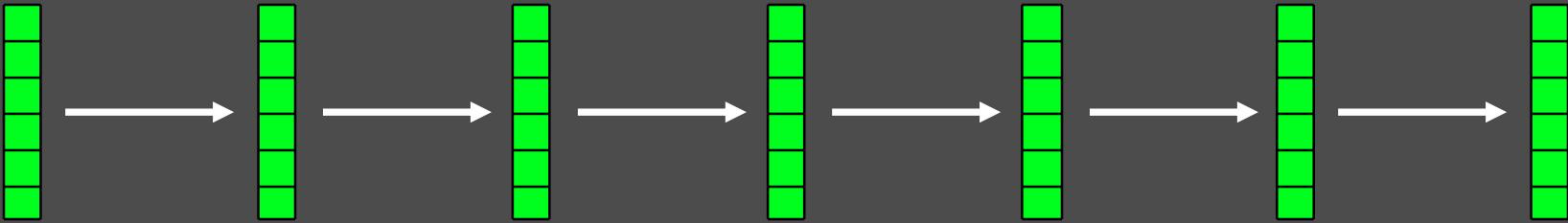
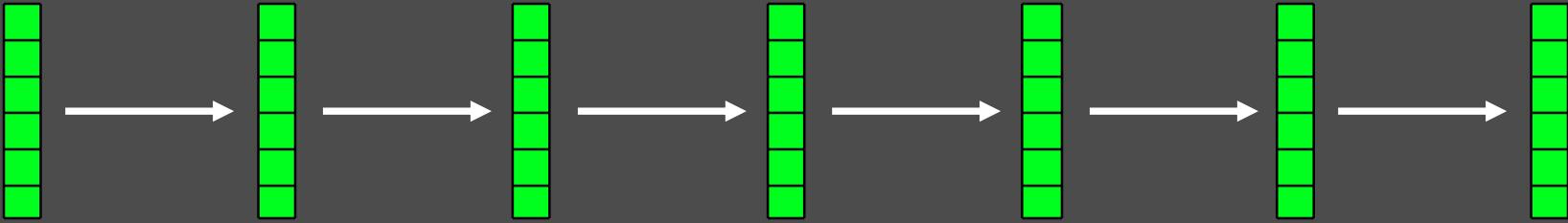
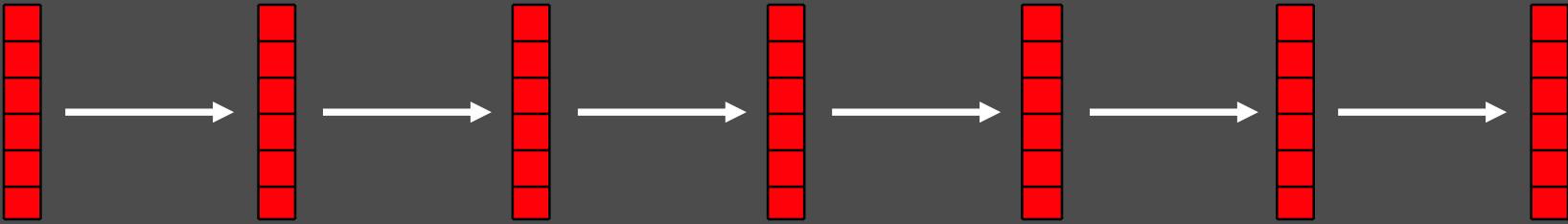
$$\min_{\mathbf{u}} \varphi(\mathbf{u}, \mathbf{Q})$$

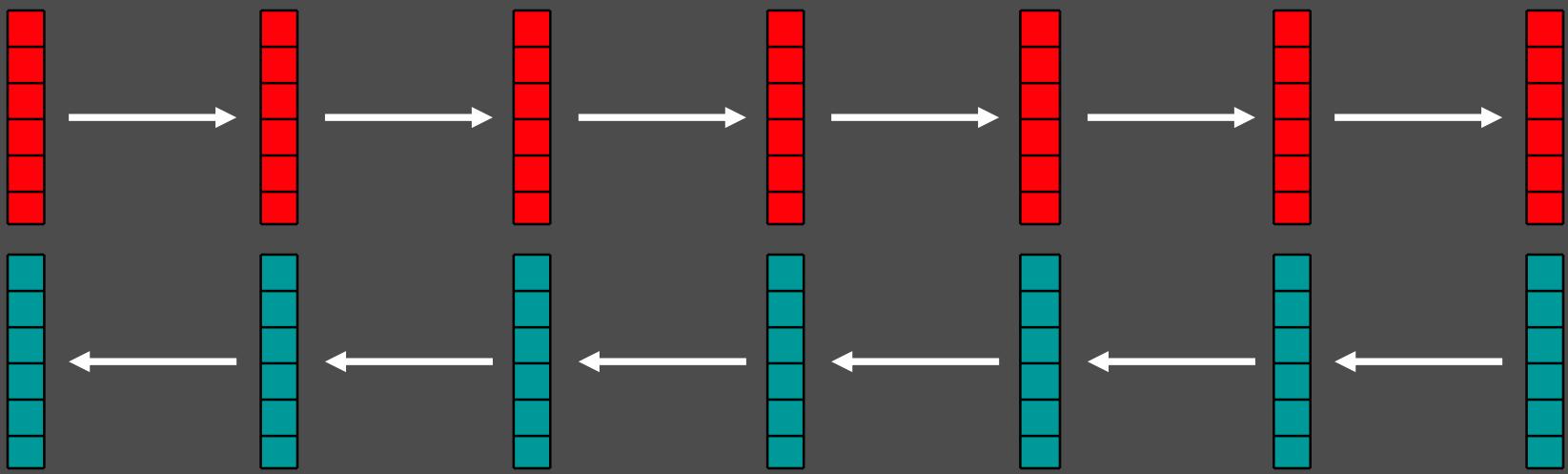


$\varphi$

$$\frac{d\varphi}{du}$$

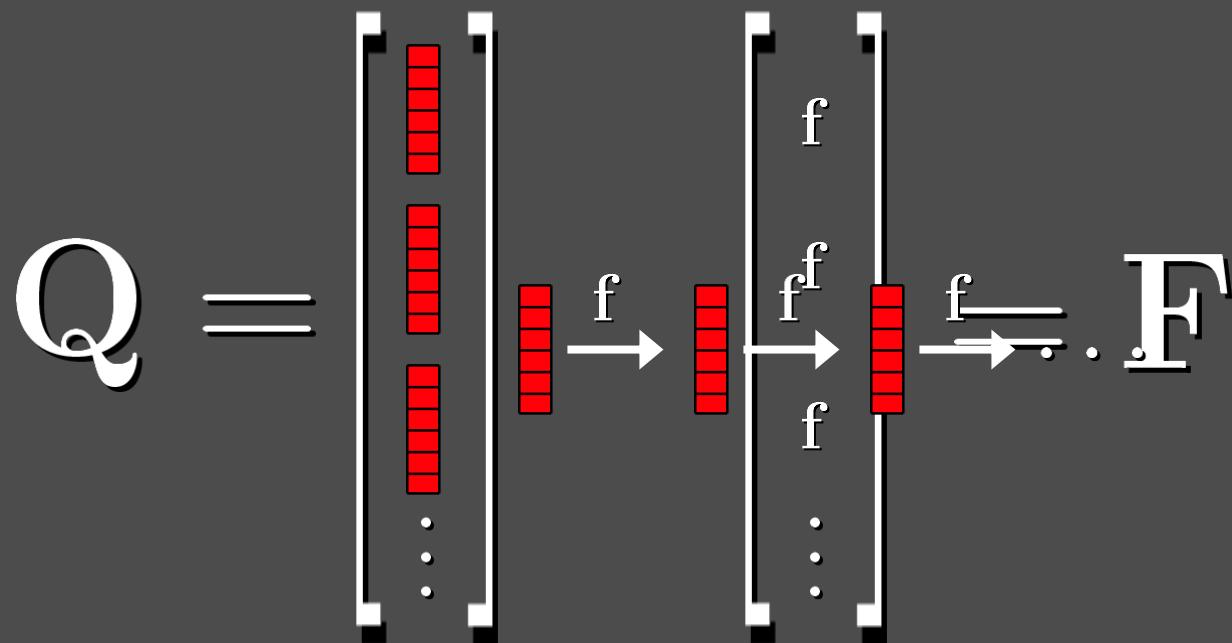
# The Adjoint Method





# The Adjoint Method

$$q_i Q_i = F_i(Q_i, u_i)$$



# The Adjoint Method

$$Q = \mathbf{E}(Q, E(Q, \psi)(Q, u))$$

$$\frac{dQ}{d\bar{Q}}$$

$$\left( {}_I d \frac{\partial F}{\partial Q} \right)$$

$$\left( \frac{\partial F}{\partial Q} \right) \frac{dQ}{du}$$

$$\frac{F}{u}$$

$$\frac{\partial F}{\partial u}$$

$$\frac{d\varphi}{d\bar{\varphi}}$$

$$\frac{d\varphi}{du} \frac{d\bar{\varphi}}{dQ} \frac{dQ}{du} +$$

$$\frac{\partial \varphi}{\partial u} \frac{dQ}{du}$$

# Duality

$$\left( I \ A \frac{\partial \mathbf{F}}{\partial \mathbf{Q}} \right)$$

$$\frac{d\mathbf{Q}}{d\mathbf{u}} =$$

$$\frac{\partial \mathbf{F}}{\partial \mathbf{u}}$$

$$\frac{\partial \varphi}{\partial \mathbf{Q}}$$

$$\frac{d\mathbf{Q}}{d\mathbf{B}}$$

$$\left( I \ -A^T \frac{\partial \mathbf{F}}{\partial \mathbf{Q}} \right)^T$$

$$\mathbf{R} =$$

$$\frac{\partial \varphi}{\partial \mathbf{Q}}^T$$

$$\mathbf{R}^T$$

$$\frac{\partial \mathbf{F}}{\partial \mathbf{u}}$$

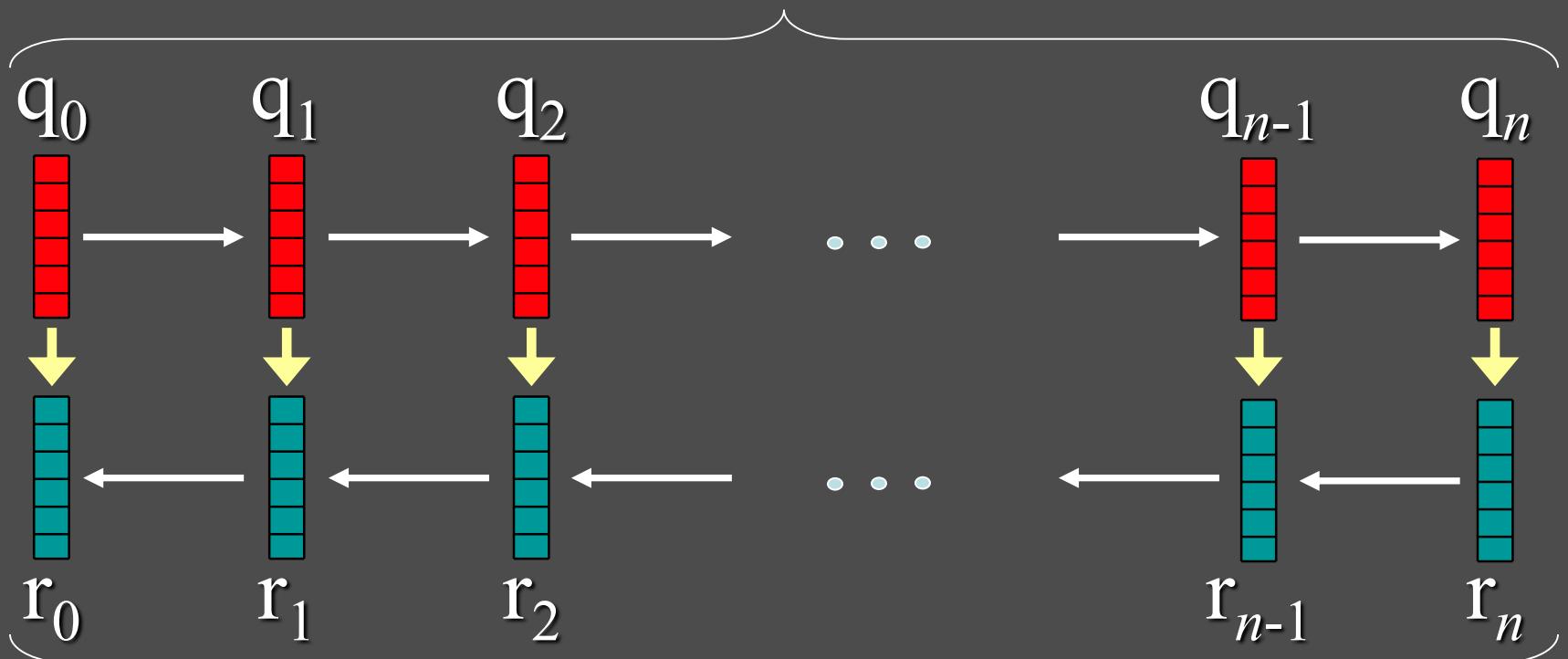
$$\boxed{\mathbf{R}^T C = \mathbf{R}^T AB = (A^T \mathbf{R})^T B = \mathbf{g}^T B}$$

# The Adjoint Method

$$\begin{aligned}
 & \text{Diagram: } \mathbf{r}_i \xrightarrow{\text{R}} \mathbf{r}_{i+1} \xrightarrow{\frac{\partial \mathbf{F}}{\partial \mathbf{Q}}} \mathbf{r}_n \\
 & \text{Equation: } \left( I - \frac{\partial \mathbf{F}}{\partial \mathbf{Q}} \right)^T \mathbf{r}_i = \left( \frac{\partial \mathbf{F}}{\partial \mathbf{Q}} \right)^T \mathbf{r}_{i+1} + \left( \frac{\partial \mathbf{F}}{\partial \mathbf{Q}} \right)^T \mathbf{r}_n
 \end{aligned}$$

# Adjoint Computation

$$\mathbf{q}_{i+1} = \mathbf{f}_i(\mathbf{q}_i, \mathbf{u})$$



$$\mathbf{r}_i = \left( \frac{\partial \mathbf{f}_i}{\partial \mathbf{q}_i} \right)^T \mathbf{r}_{i+1} + \left( \frac{\partial \varphi}{\partial \mathbf{q}_i} \right)^T$$

# The Gradient

$\mathbf{R}^T$

$$\frac{\partial \mathbf{F}}{\partial \mathbf{u}} \mathbf{R}^T \frac{\partial \mathbf{F}}{\partial \mathbf{u}} = \frac{\partial \varphi}{\partial \mathbf{Q}} \frac{d\mathbf{Q}}{d\mathbf{u}} \frac{\partial \varphi}{\partial \mathbf{Q}}$$

$$\frac{d\mathbf{Q}}{d\mathbf{u}}$$

$\mathbf{r}_0$

$\mathbf{r}_1$

$\mathbf{r}_2$

$\mathbf{r}_{n-1}$

$\mathbf{r}_n$

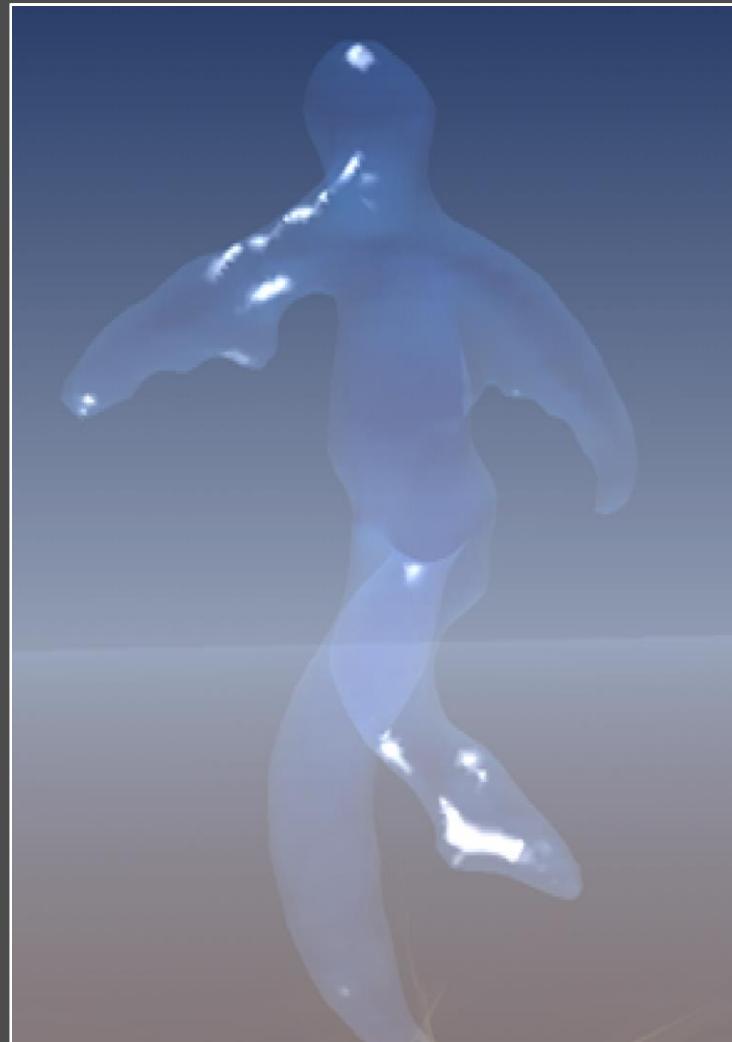
# The Gradient

$$\mathbf{R}^T \frac{\partial \mathbf{F}}{\partial \mathbf{u}} = \frac{\partial \varphi}{\partial \mathbf{Q}} \frac{d\mathbf{Q}}{d\mathbf{u}}$$

$$\frac{d\varphi}{d\mathbf{u}} = \mathbf{R}^T \frac{\partial \mathbf{F}}{\partial \mathbf{u}} + \frac{\partial \varphi}{\partial \mathbf{u}}$$

# Adjoint Method Summary

- Two Passes
  - $\neq$  number of controls
  - High memory cost
- Very Fast
  - Complex Animations!



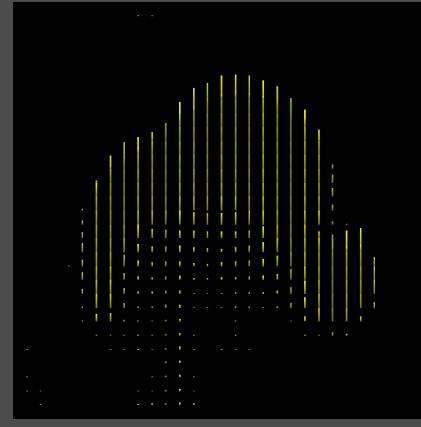
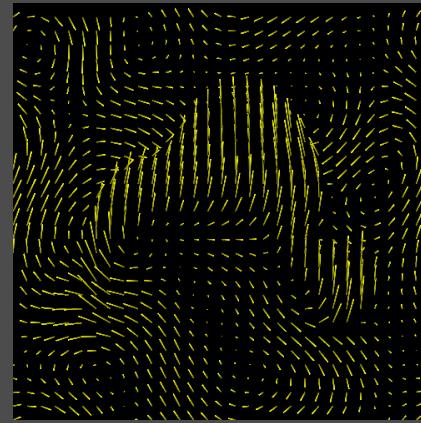
# Adjoint Fluid Control

# Adjoint Steps

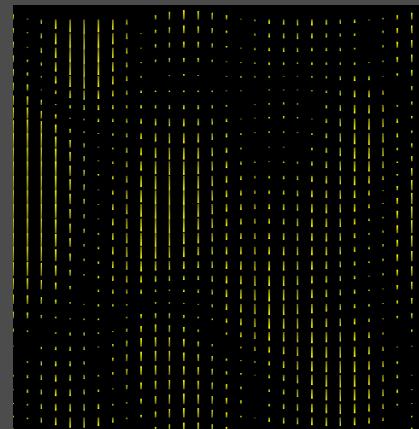
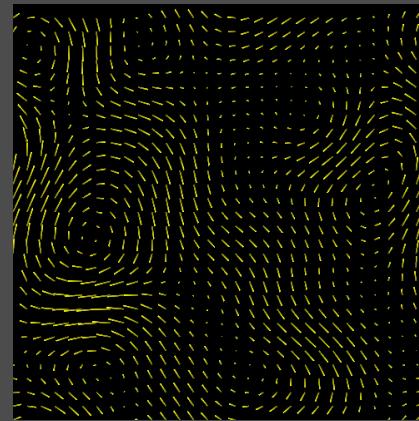
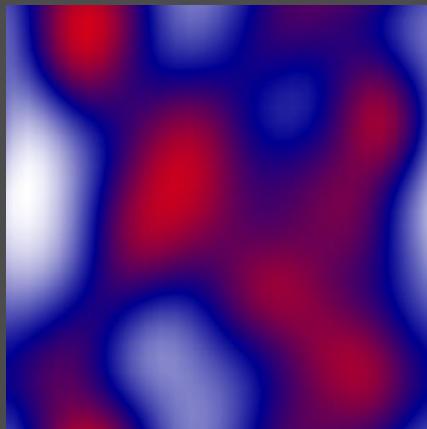
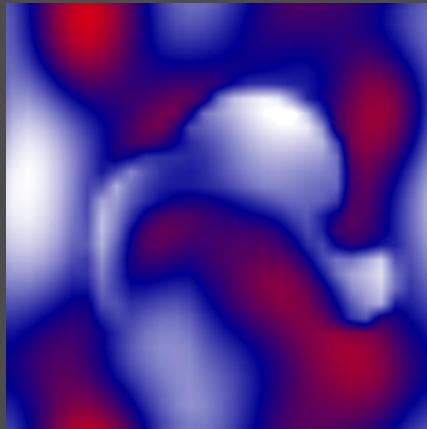
$$\mathbf{r}_i = \left( \frac{\partial \mathbf{f}_i}{\partial \mathbf{q}_i} \right)^T \mathbf{r}_{i+1}$$

- Heat
- Projection/Diffusion
- Advection
- Fast-Marching

# Heat

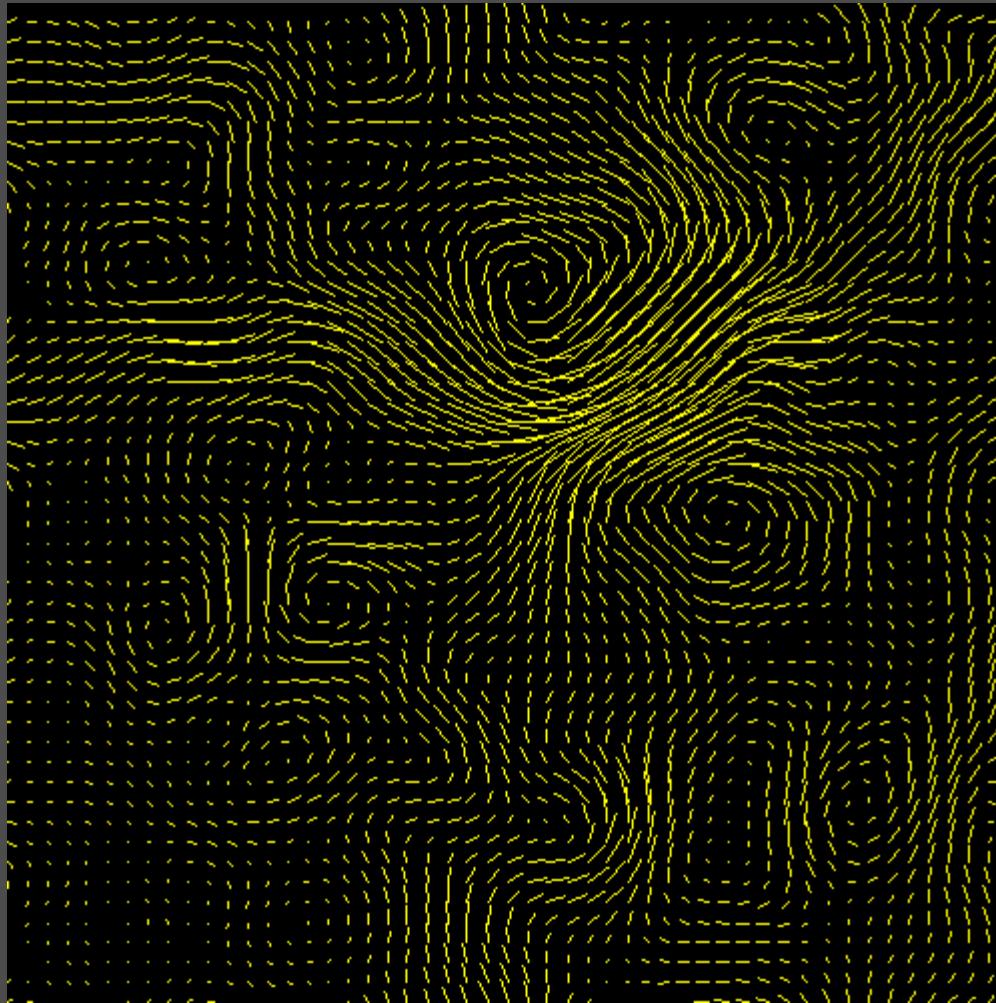


# *Adjoint Heat*



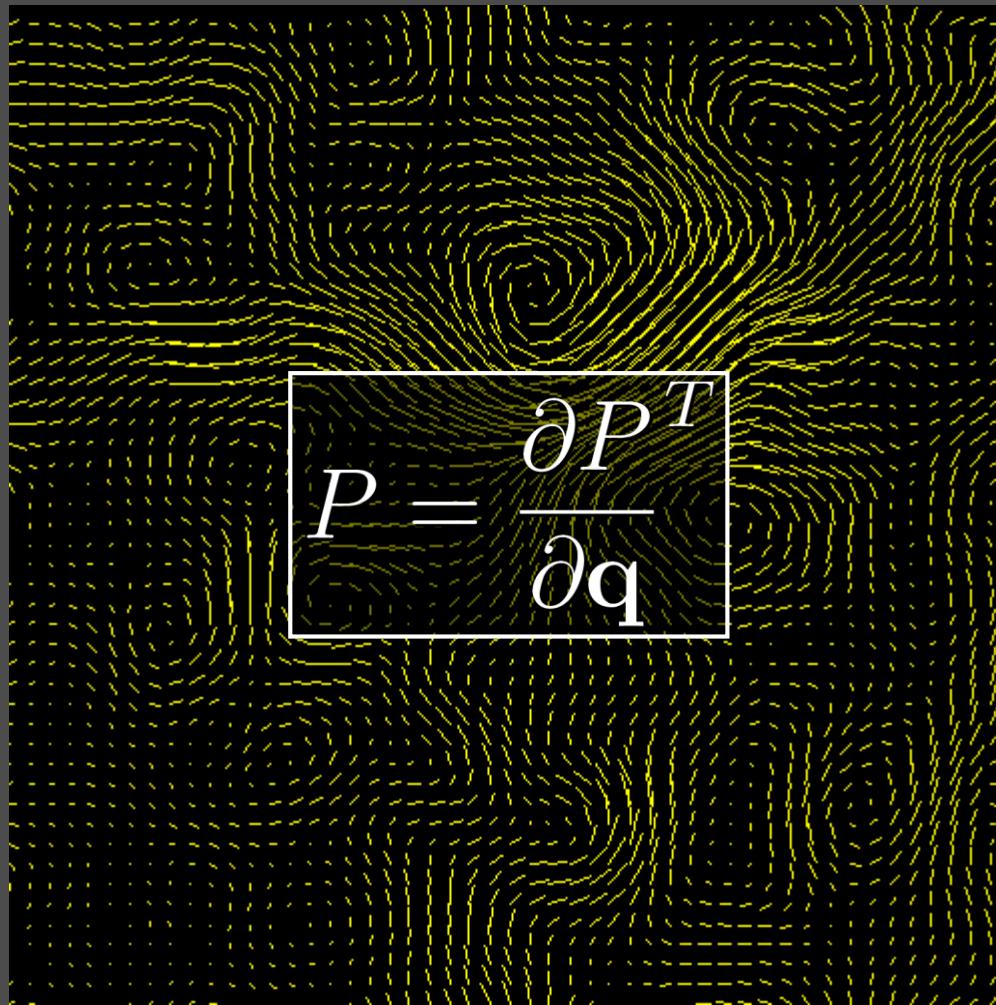
# Projection

P



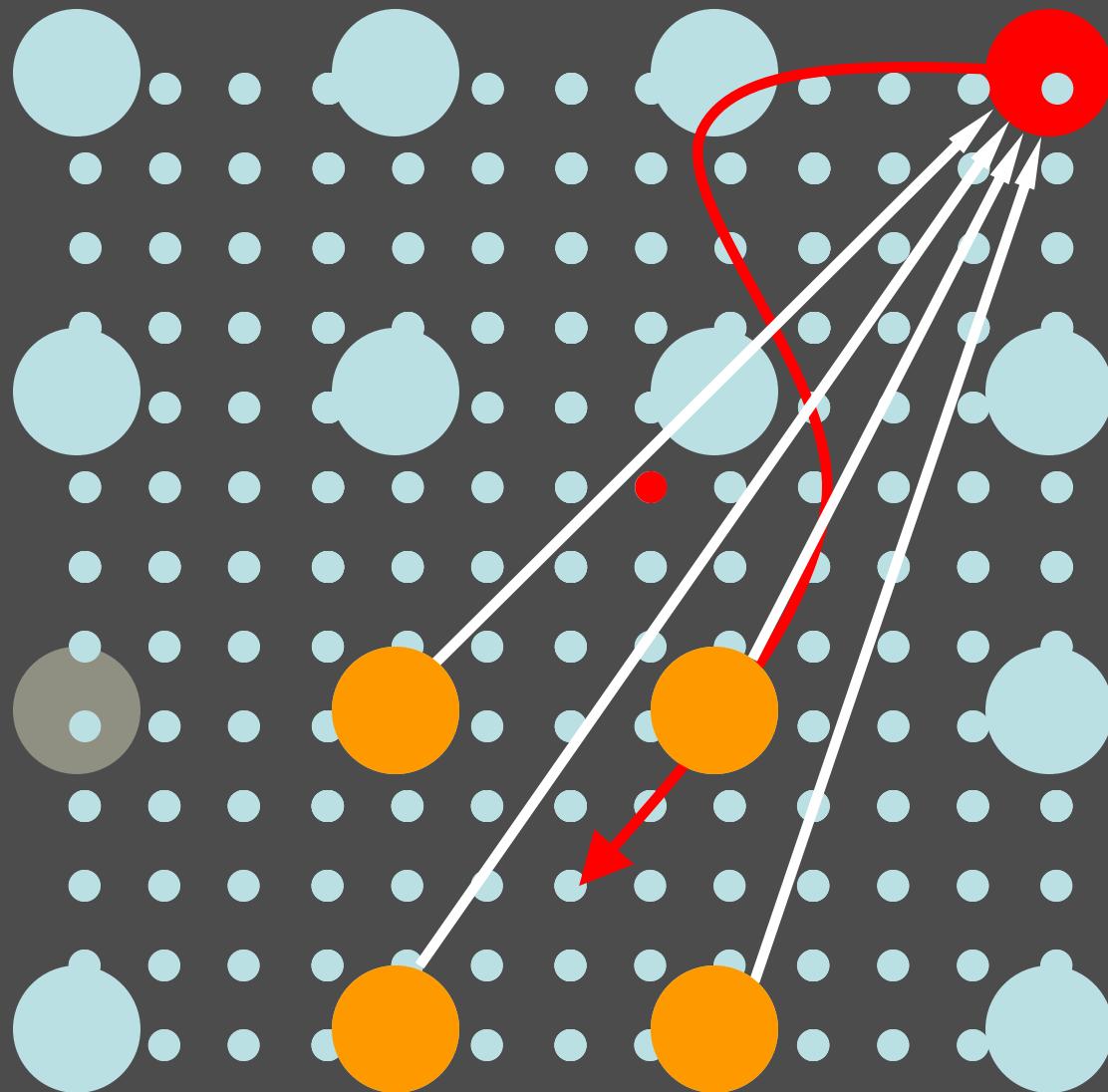
# *Adjoint Projection*

$$\frac{\partial P^T}{\partial \mathbf{q}}$$

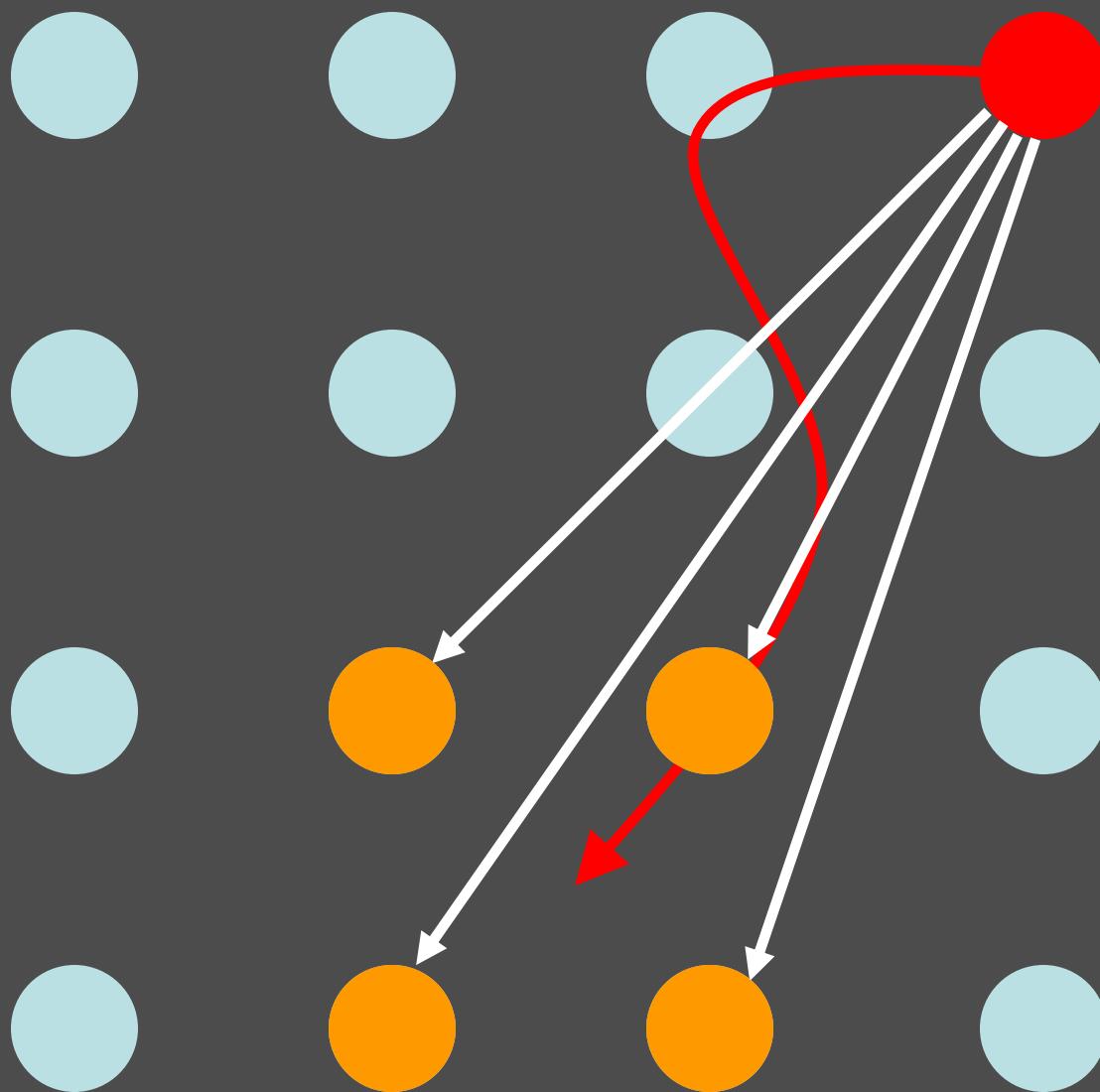


- Same goes for diffusion!

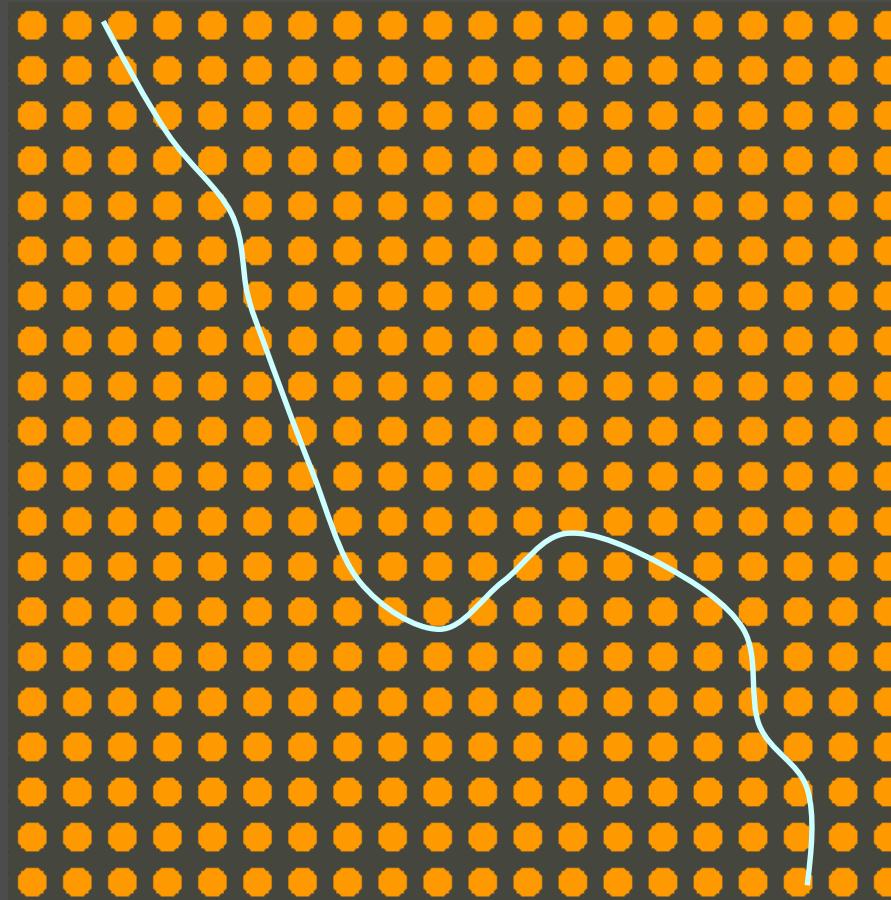
# Advection



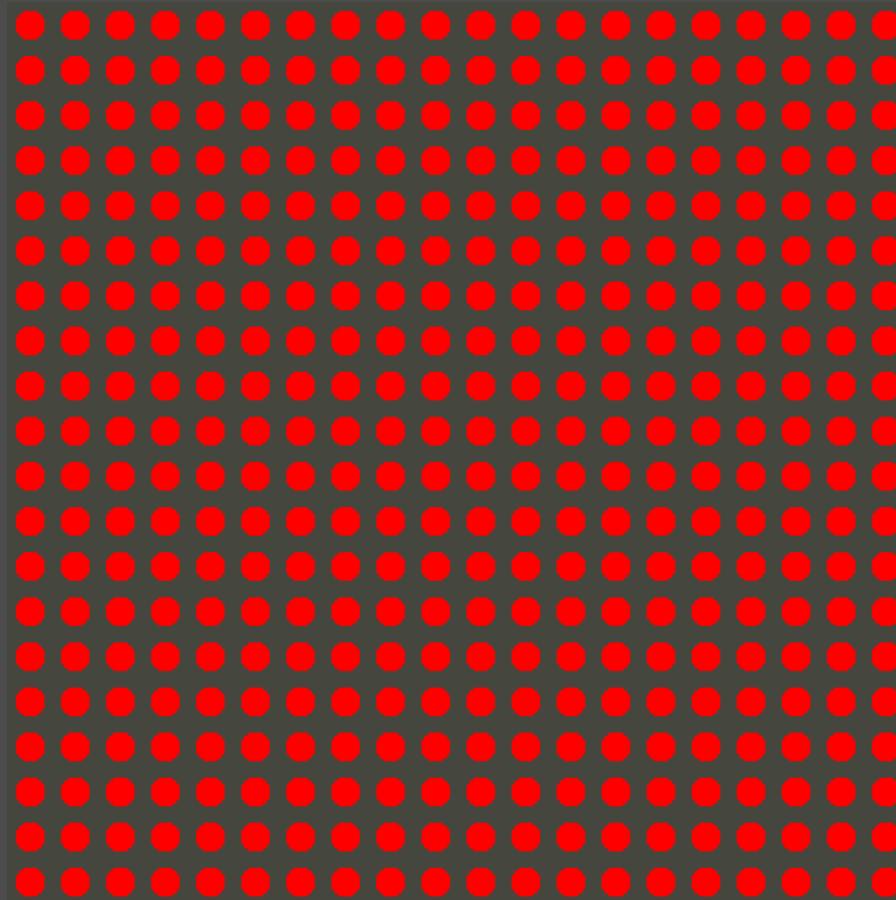
# *Adjoint Advection*



# Fast Marching



# *Adjoint Fast Marching*



# Results





Grid: 50×50×50

Timesteps:15

Controls: 100,000

Memory: 600MB



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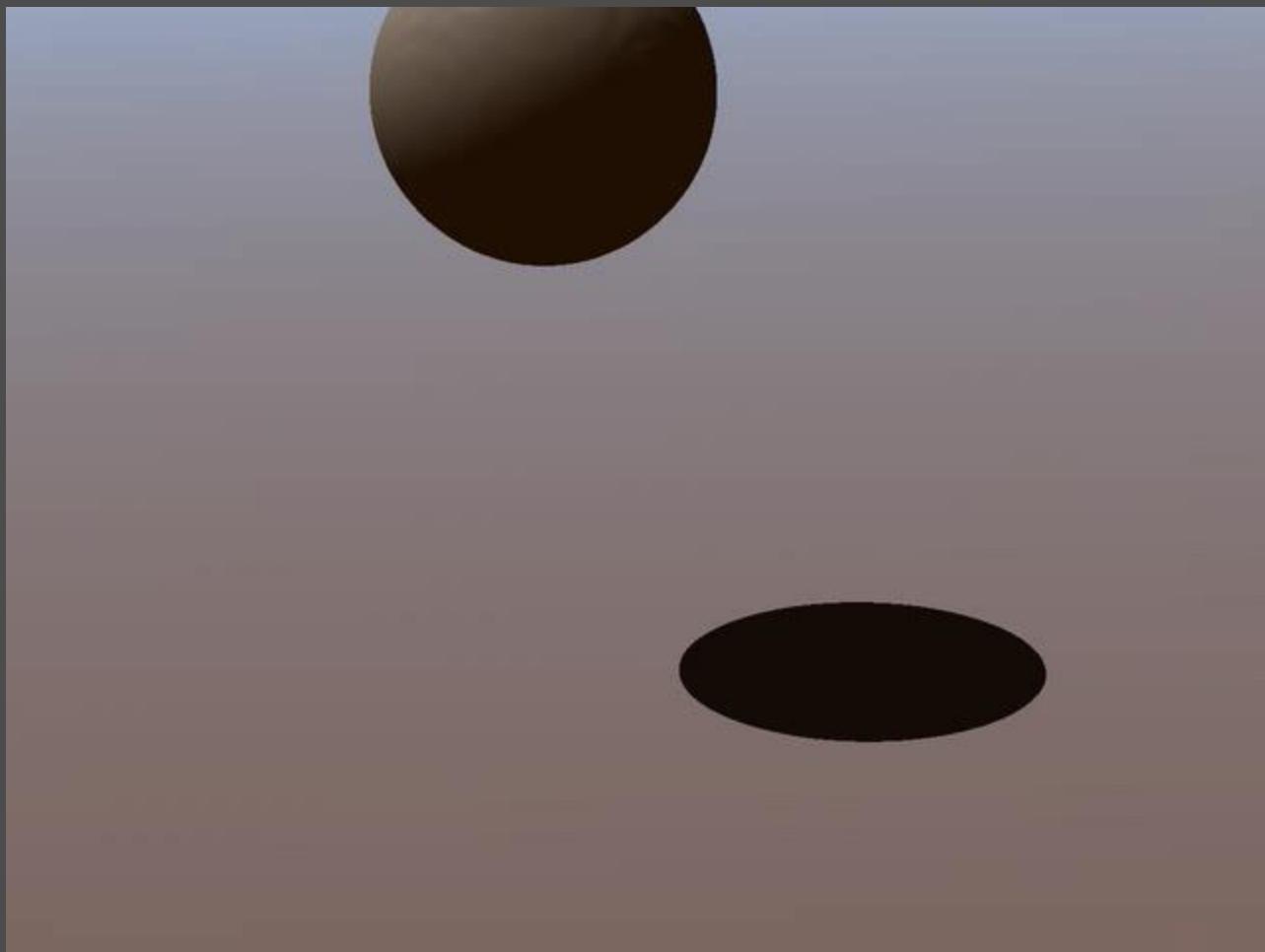


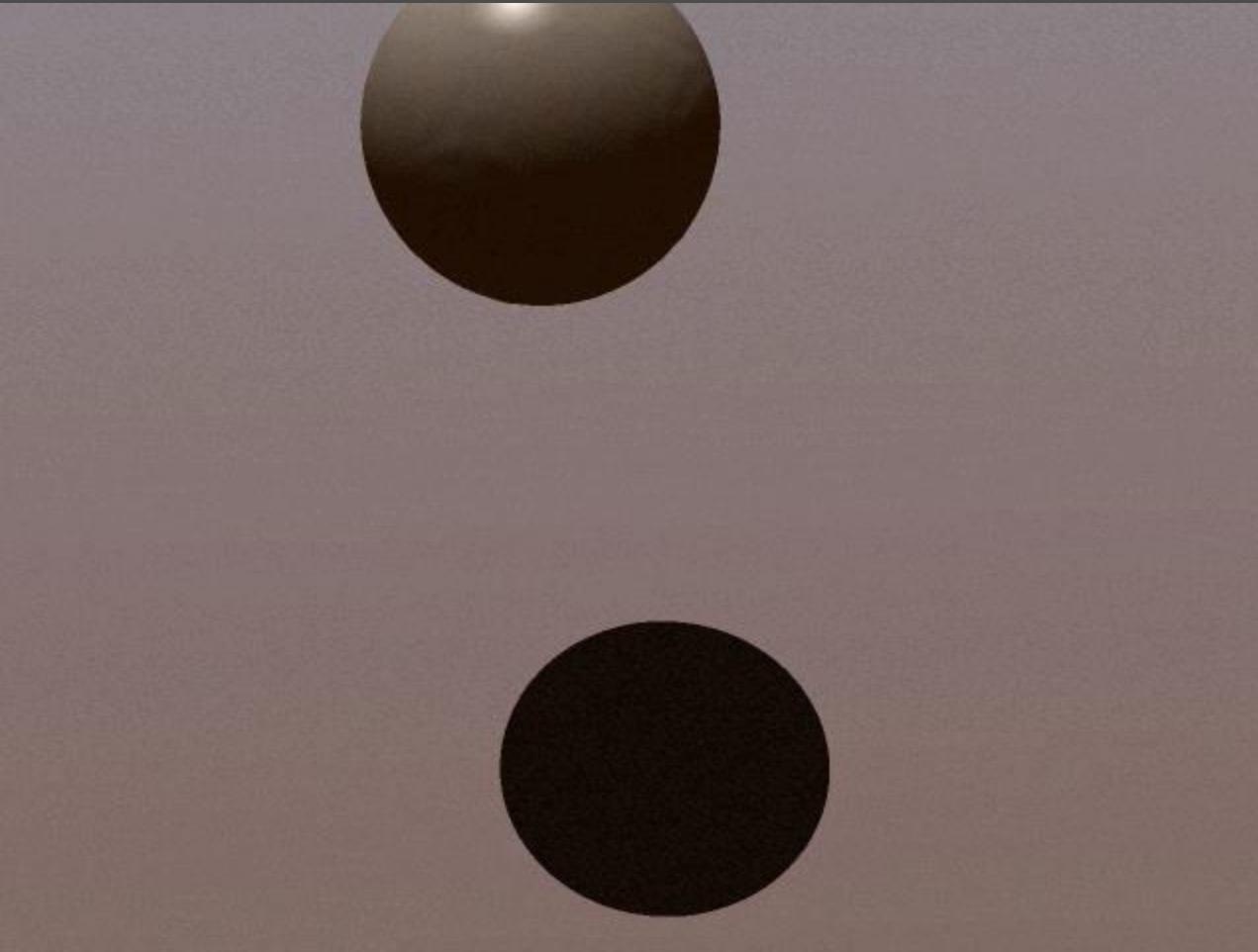
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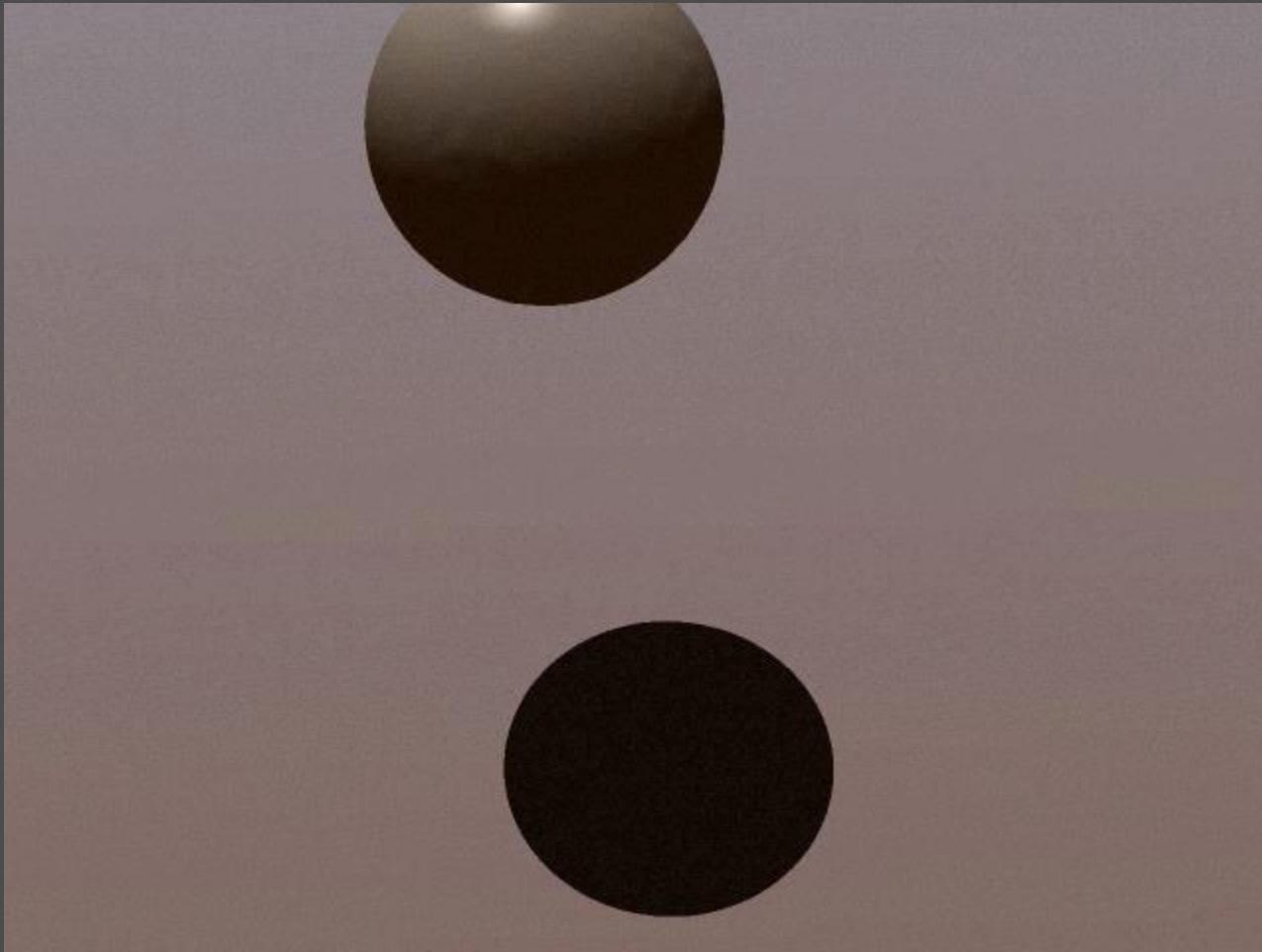


Grid: 50×23×50

Timesteps:10

Controls: 300,000

Memory: 200MB

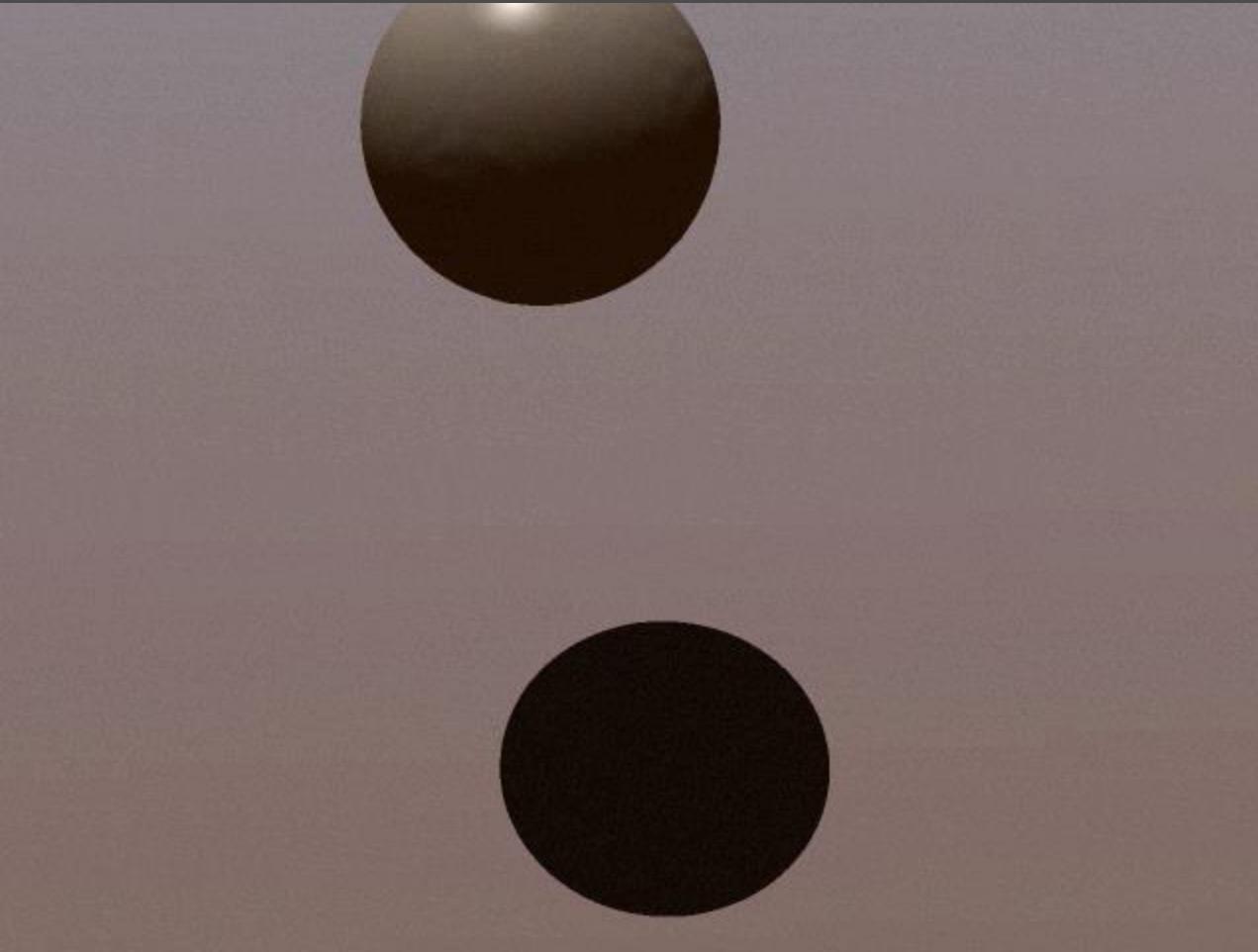


Grid: 50×23×50

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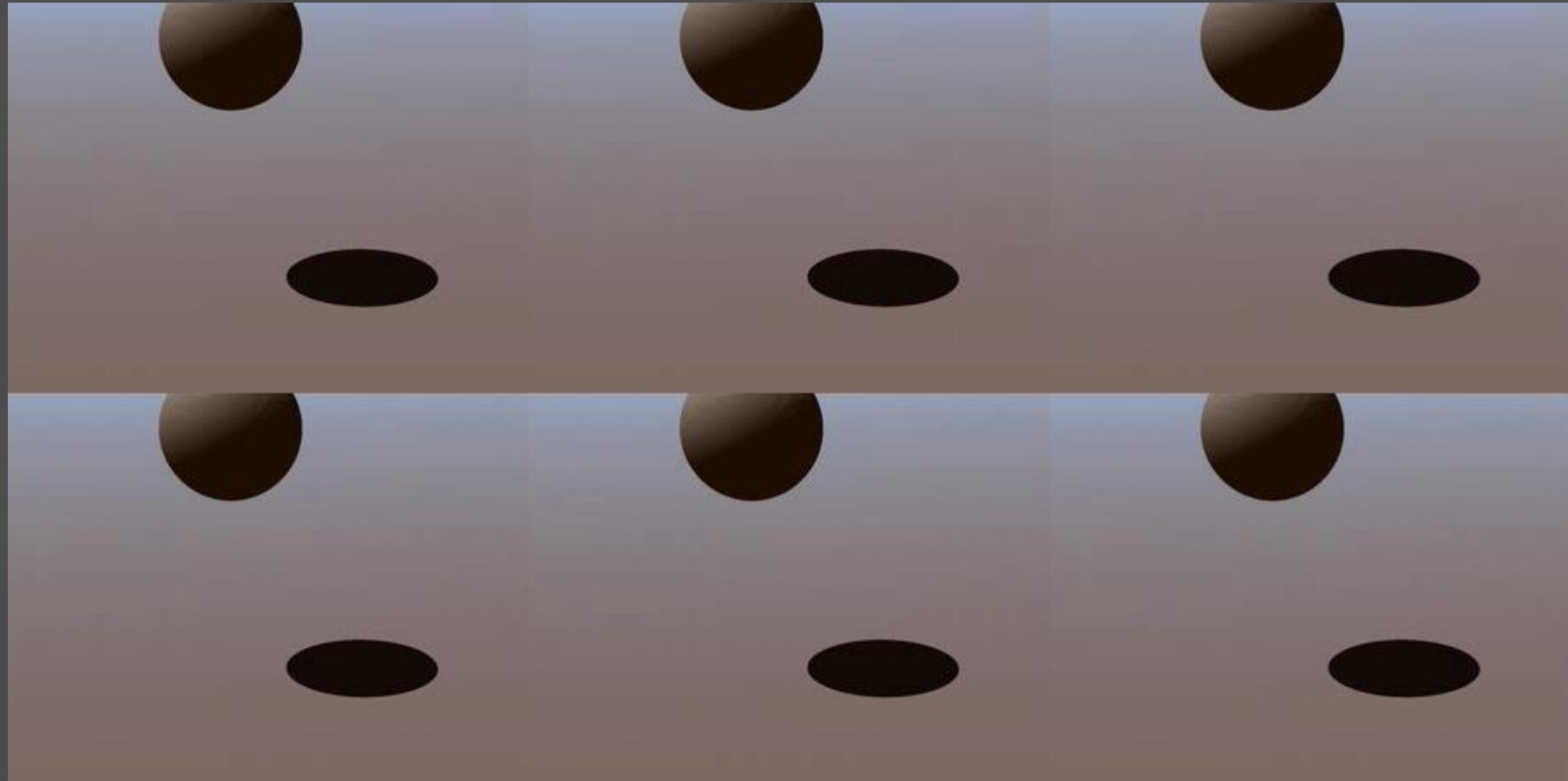


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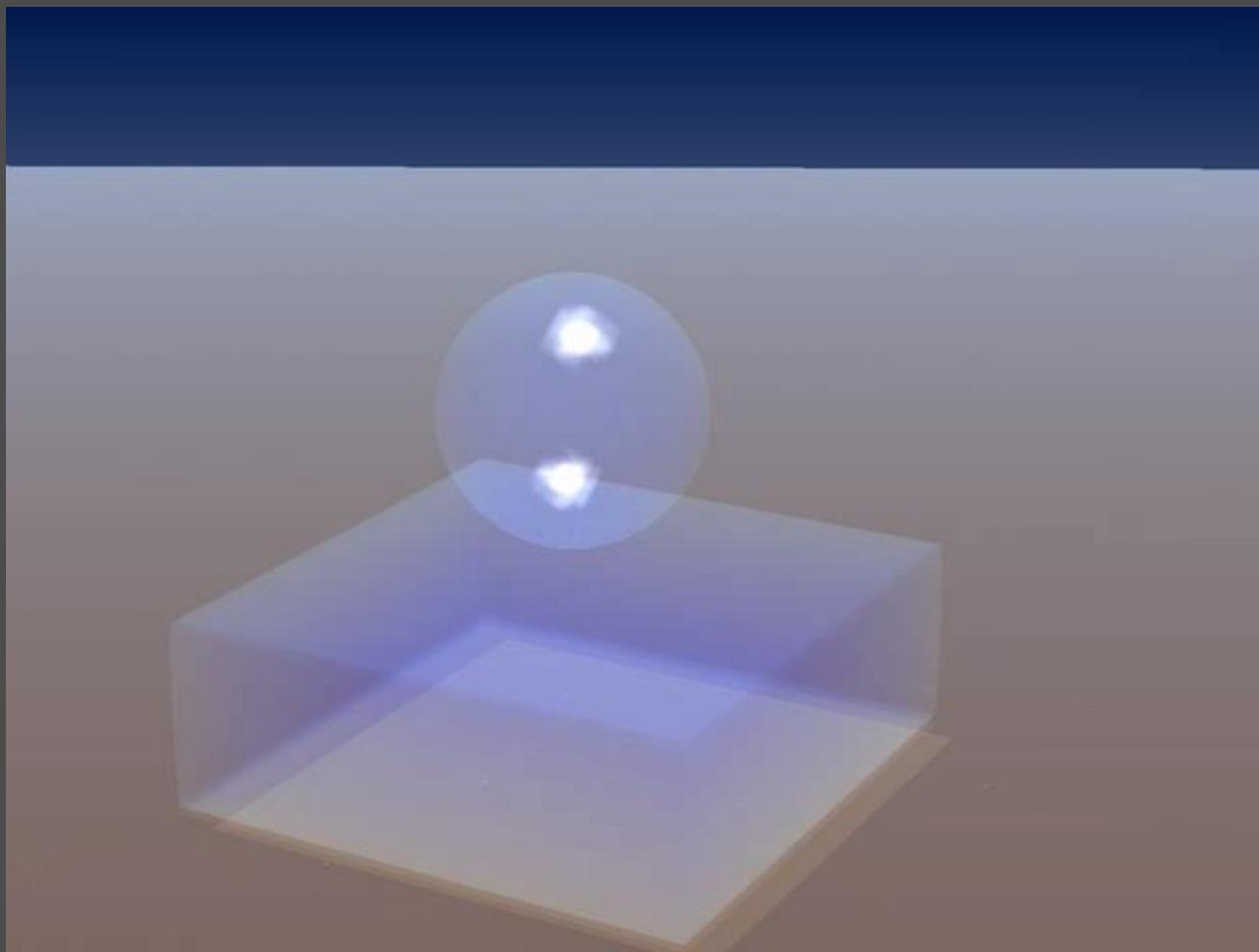


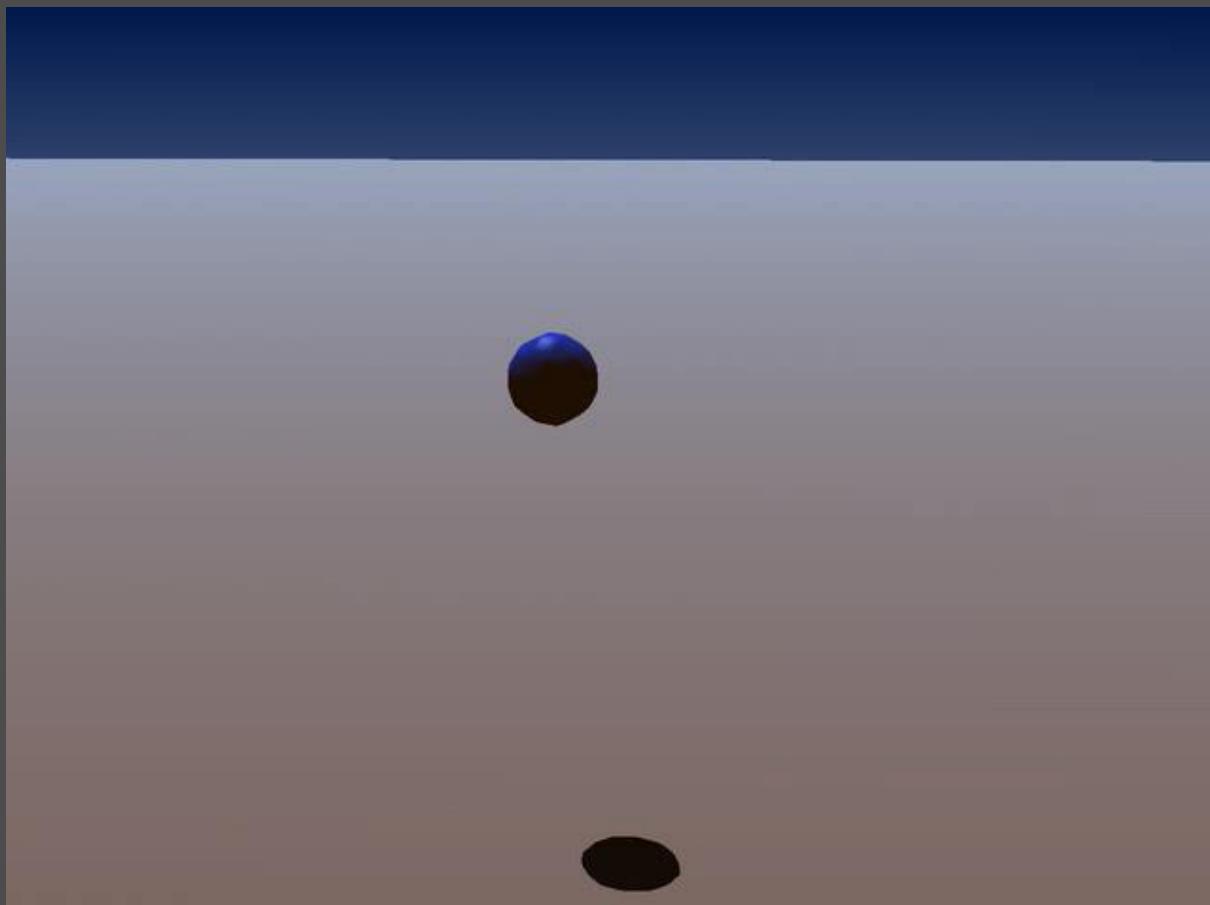
Grid: 50×23×50

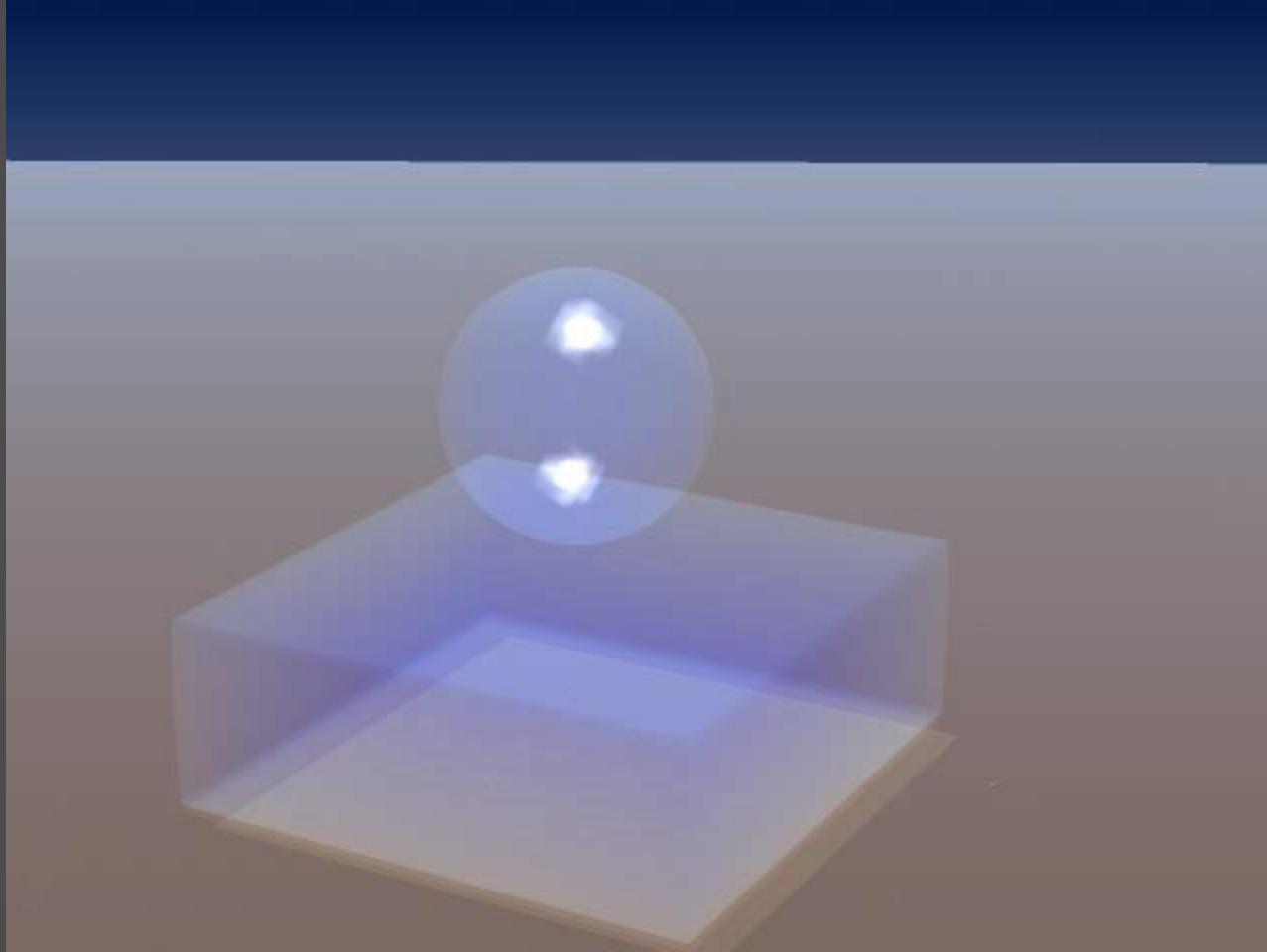
Timesteps:10

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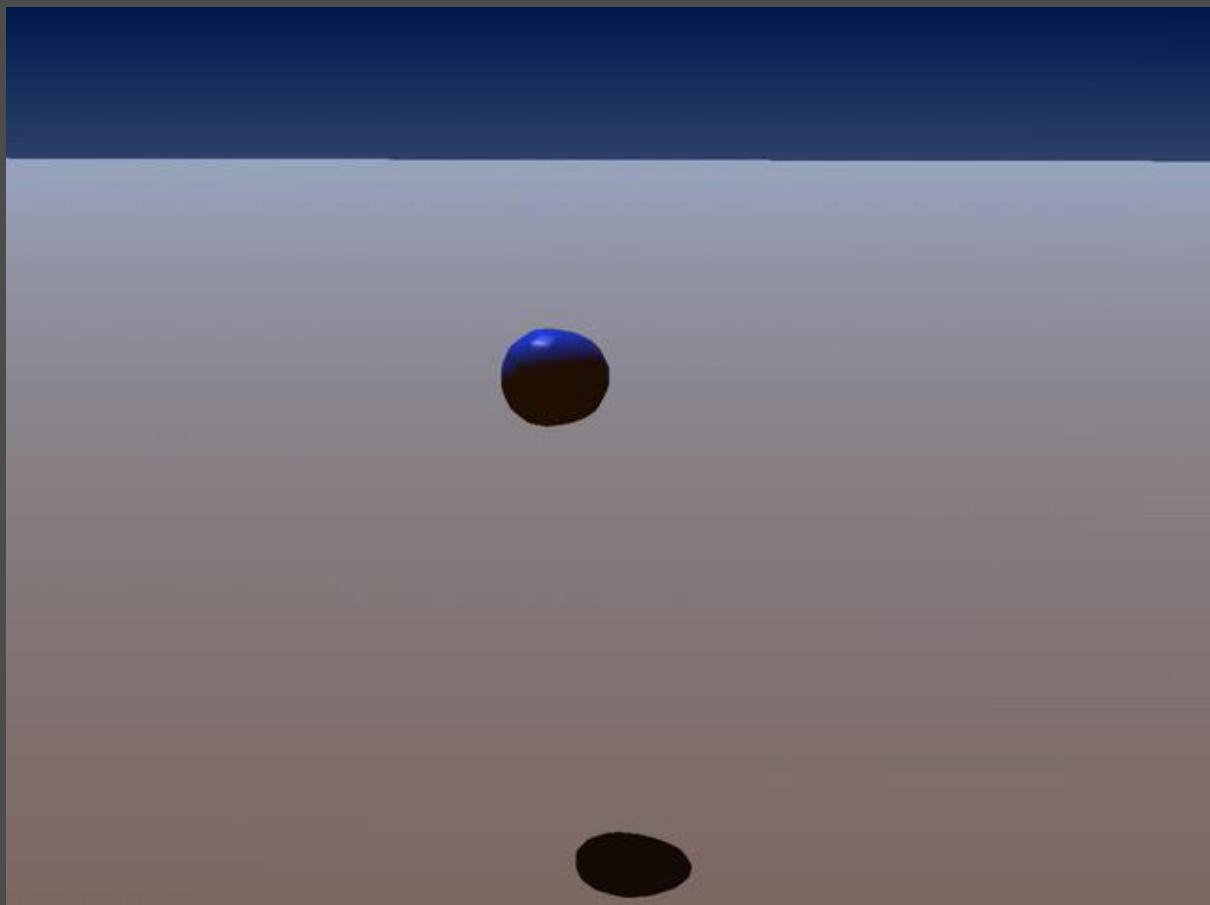


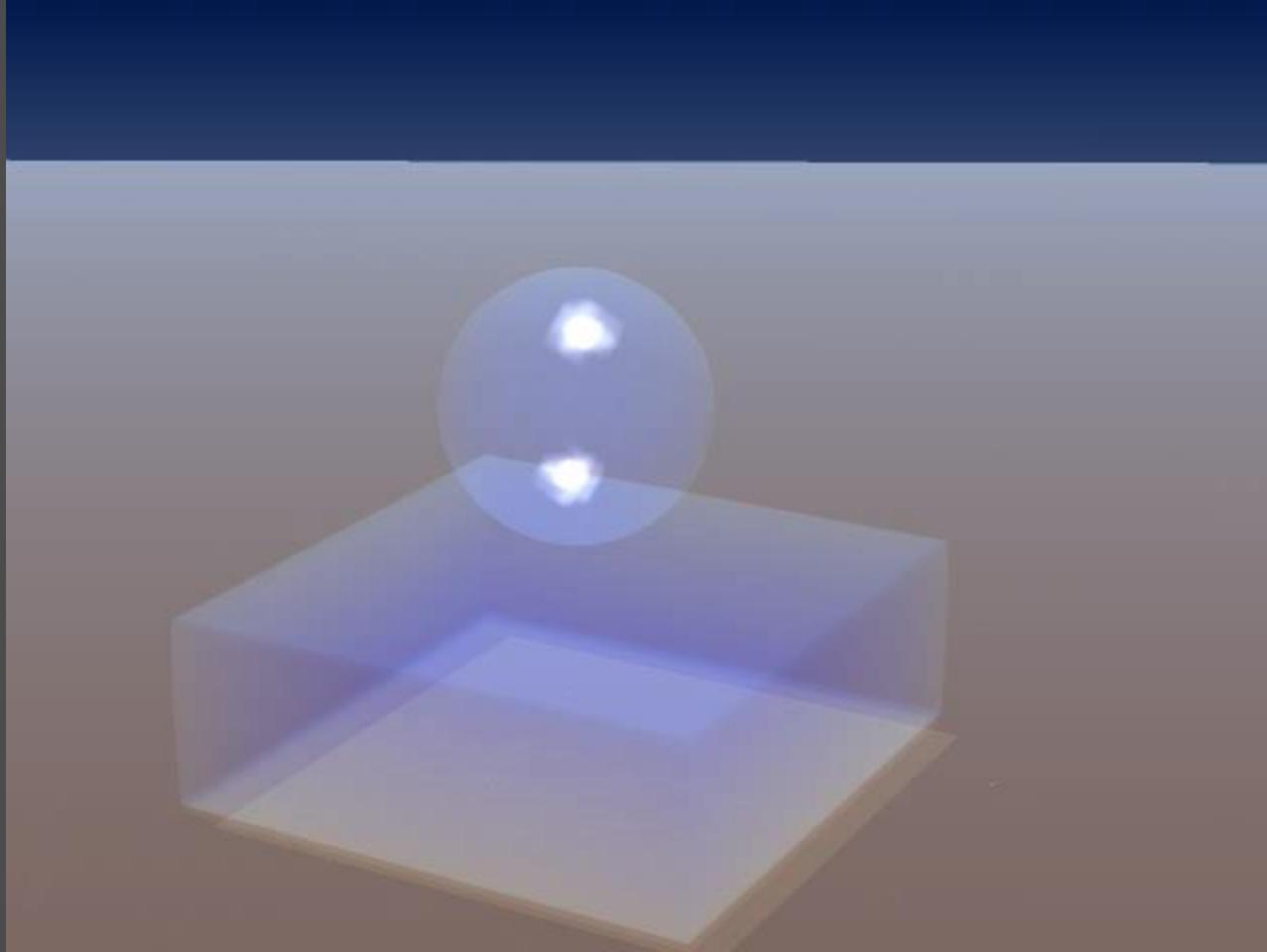


Grid: 30×30×30

Timesteps:10

Memory: 200MB

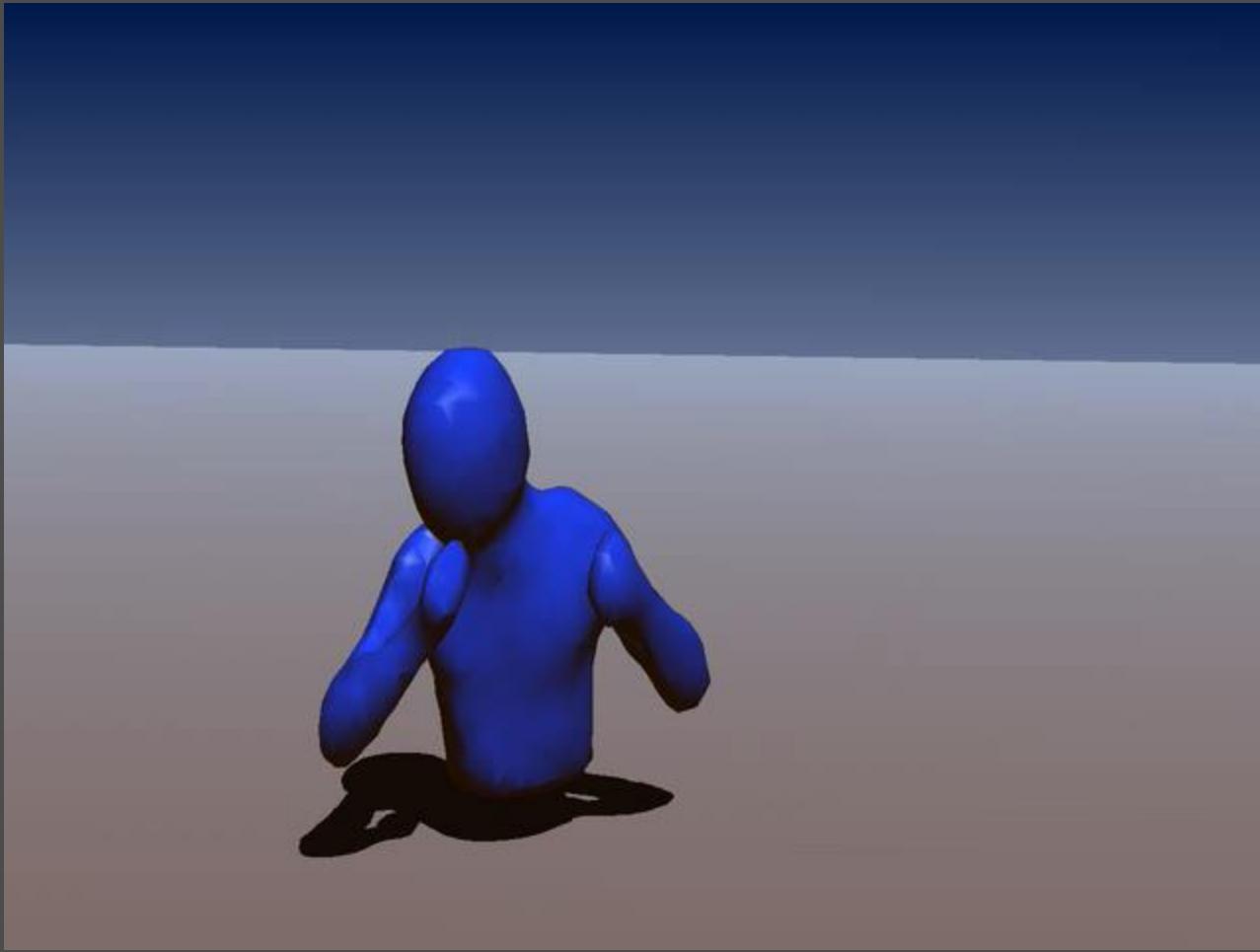


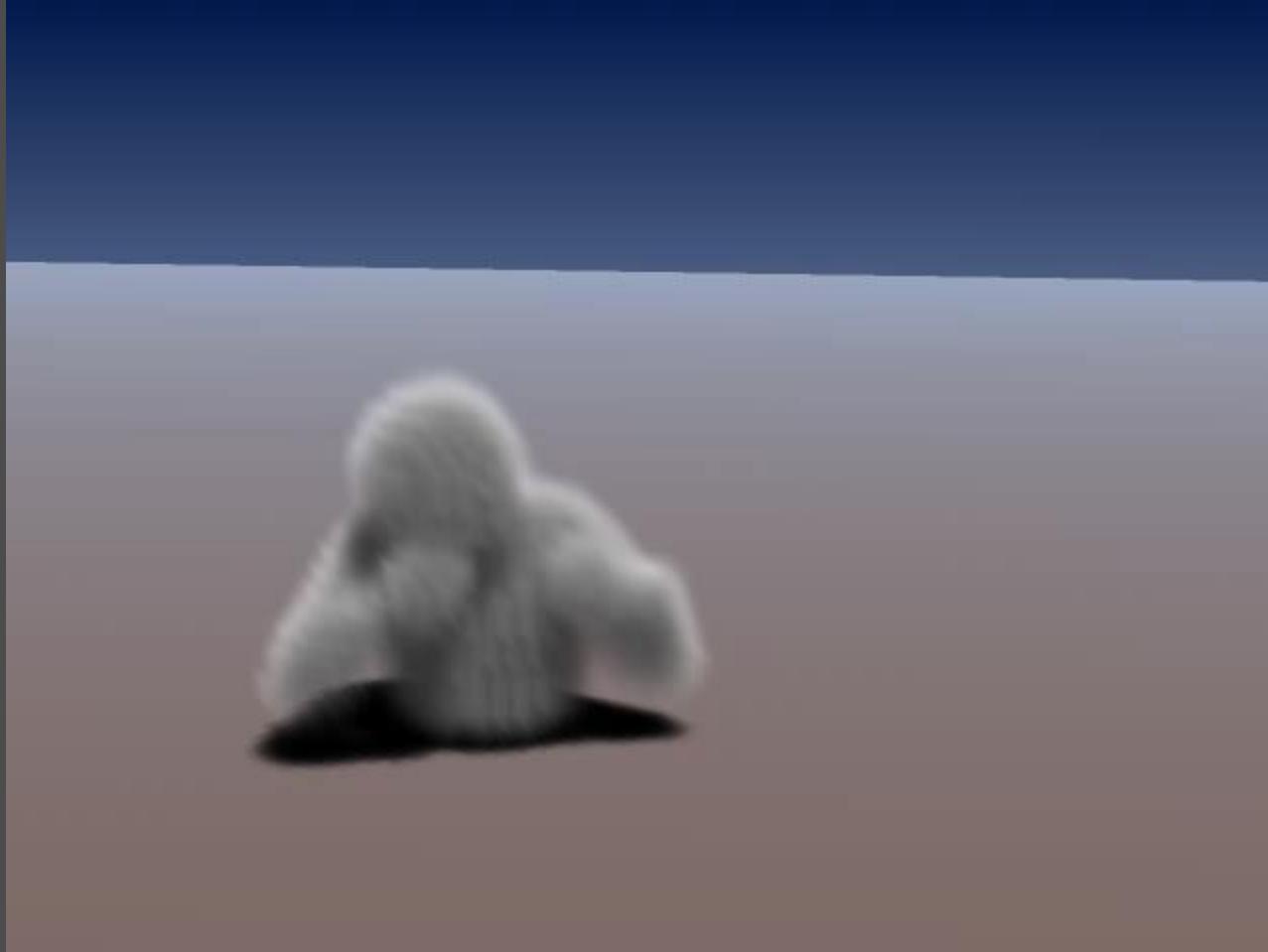


Grid: 30×30×30

Timesteps:10

Memory: 200MB



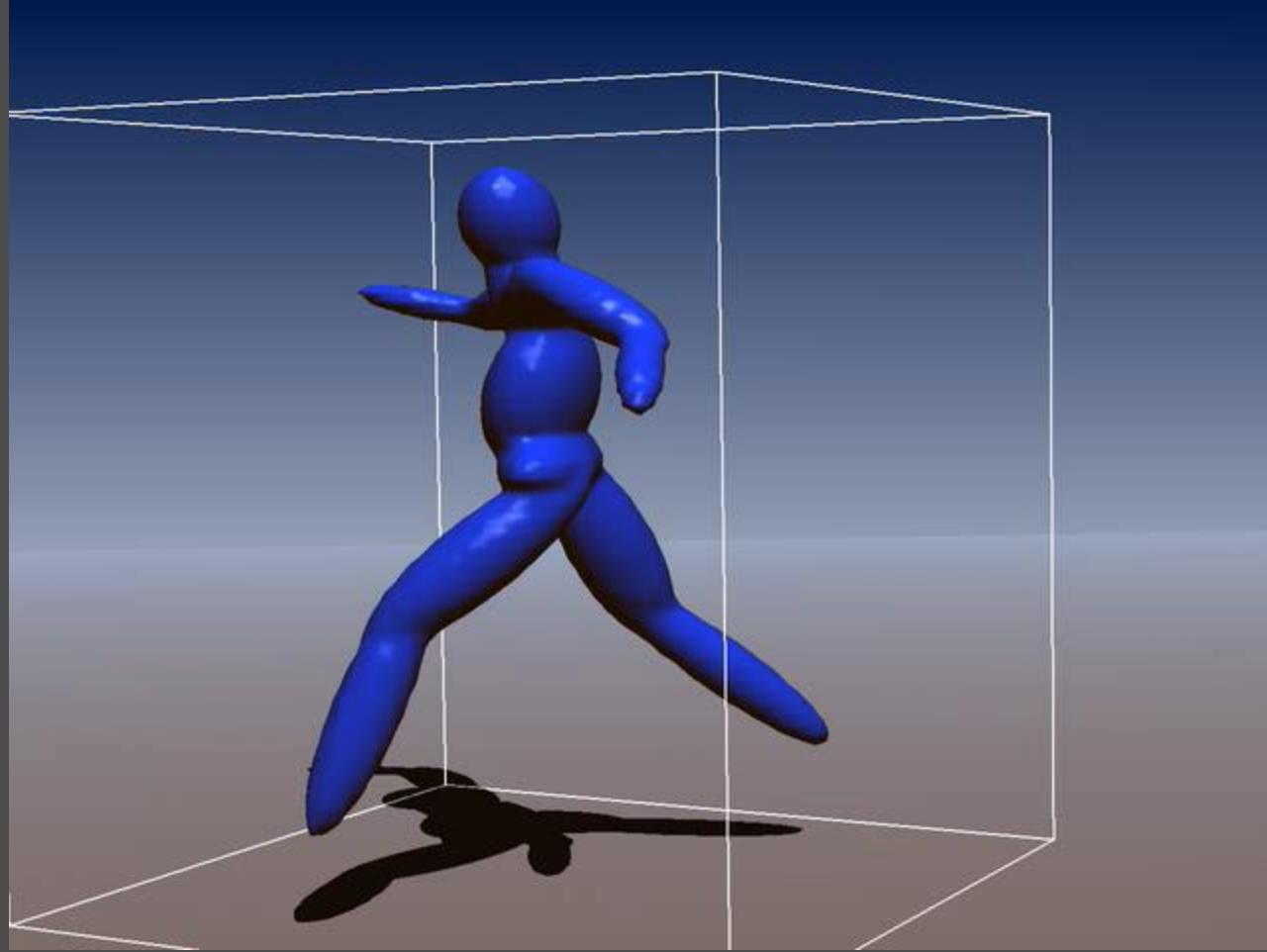


Grid: ?x?x?

Timesteps:20

Controls: 600,000

Memory: 550MB



Grid: 45×50×36

Timesteps: 46

Controls: 1.5 Million

Memory: 600MB



Grid: 45×50×36

Timesteps: 46

Controls: 1.5 Million

Memory: 600MB



Grid: 45×50×36

Timesteps: 46

Controls: 1.5 Million

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Timesteps: 46

Controls: 1.5 Million

Memory: 600MB



Grid: 45×50×36

Timesteps: 46

Controls: 1.5 Million

Memory: 600MB

# Contributions

- High-level control of fluid simulations
  - Smoke
  - Clay
  - Water
- Free surface fluids derivatives
  - Fast marching
- Adjoint method

# Thanks

- John Anderson, Peter Schmid, Donna Calhoun
- UW Animation Research Labs
- NSF grant CCR-0092970, ITR grant IIS-0113007
- Alfred P. Sloan Fellowship
- NSF Graduate Research Fellowship
- Alias|Wavefront, Microsoft Research, Intel, EA, Sony

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