Stable Fluids

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Fluids in Computer Graphics

- Interactive modeling of fluids
- Fast
- Capture visuals of fluids

Fluid Mechanics

- Natural framework for fluid modeling

  Full Navier-Stokes Equations
- Has a long history

  reuse code/algorithms
- Equations are hard to solve

  non-linear

Previous Work (computer graphics)

Two dimensions:

- Yaeger & Upson 86 + Gamito et al. 95 (vortex blobs)
- Chen et al. 97 (explicit in time, finite differences)

Three-dimensions:

- Foster & Metaxas 97 (explicit in time, finite differences)

unstable

Inaccurate schemes can be useful

Main Contribution

Stable Navier-Stokes solver

Any time step can be used

Bigger time steps = faster simulations

NOT accurate

Application

Use velocity to move densities:

While (simulating)
- Get force from UI
- Get density from UI
- Update velocity
- Update density
- Display density

Equations

$$\frac{\partial \rho}{\partial t} = -(\mathbf{u} \cdot \nabla)\rho + \kappa \nabla^2 \rho + S$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

+ velocity should conserve mass

Equations very similar

Equations

Evolution of density (assume velocity known)

$$\boxed{\frac{\partial \rho}{\partial t}} = -(\mathbf{u} \cdot \nabla)\rho + \kappa \nabla^2 \rho + S$$

Over a time step...

Equations

Evolution of density (assume velocity known)

$$\frac{\partial \rho}{\partial t} = -(\mathbf{u} \cdot \nabla)\rho + \kappa \nabla^2 \rho + S$$

Density changes in the direction of the flow

Equations

Evolution of density (assume velocity known)

$$\frac{\partial \rho}{\partial t} = -(\mathbf{u} \cdot \nabla)\rho + \kappa \nabla^2 \rho + S$$

Density diffuses over time

Equations

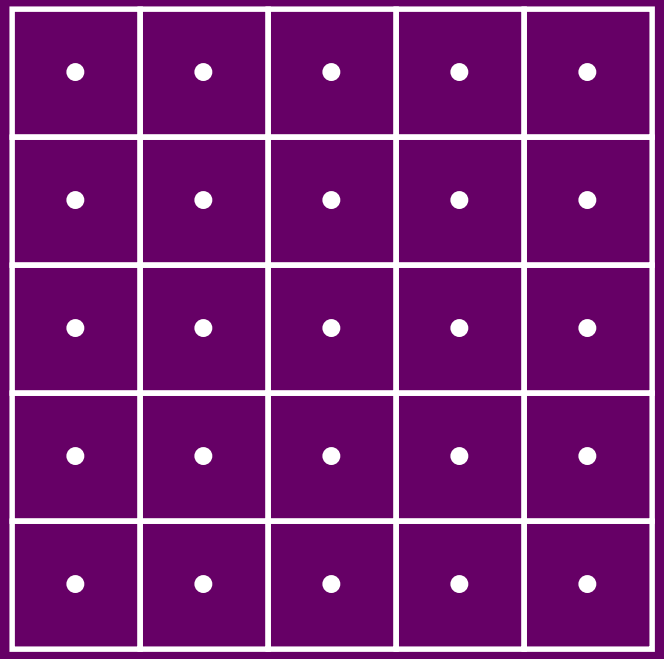
Evolution of density (assume velocity known)

$$\frac{\partial \rho}{\partial t} = -(\mathbf{u} \cdot \nabla)\rho + \kappa \nabla^2 \rho + \boxed{S}$$

Increases due to sources from the UI

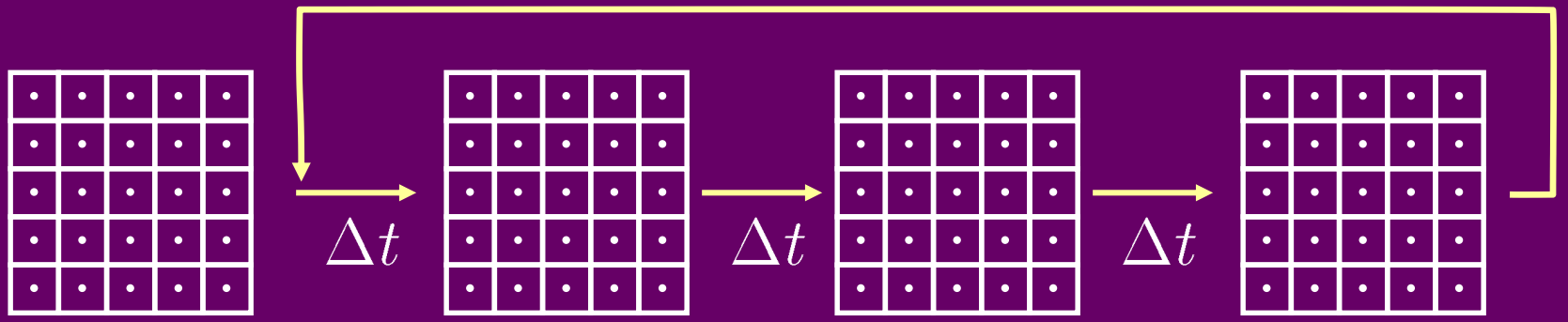
Algorithm

Subdivide space into voxels



Velocity + density defined in the center of each voxel

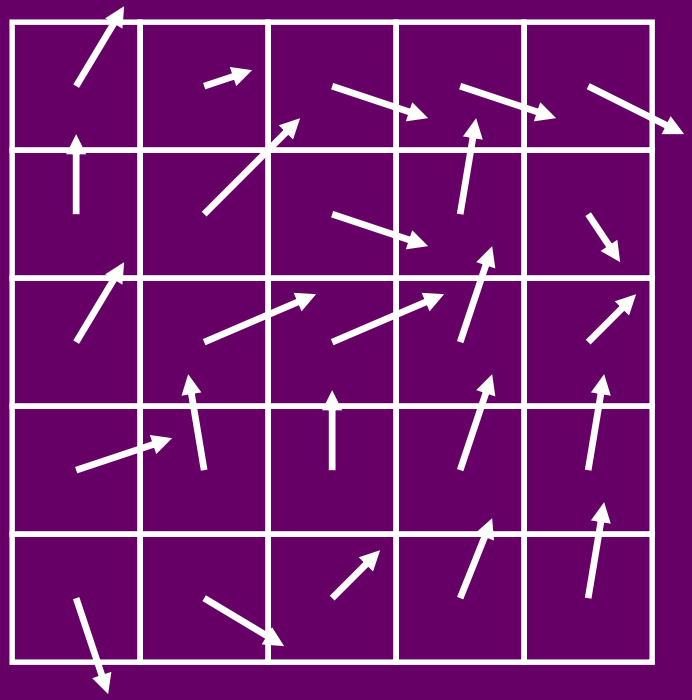
Algorithm



add source diffuse move

$$\frac{\partial \rho}{\partial t} = -(\mathbf{u} \cdot \nabla)\rho + \kappa \nabla^2 \rho + S$$

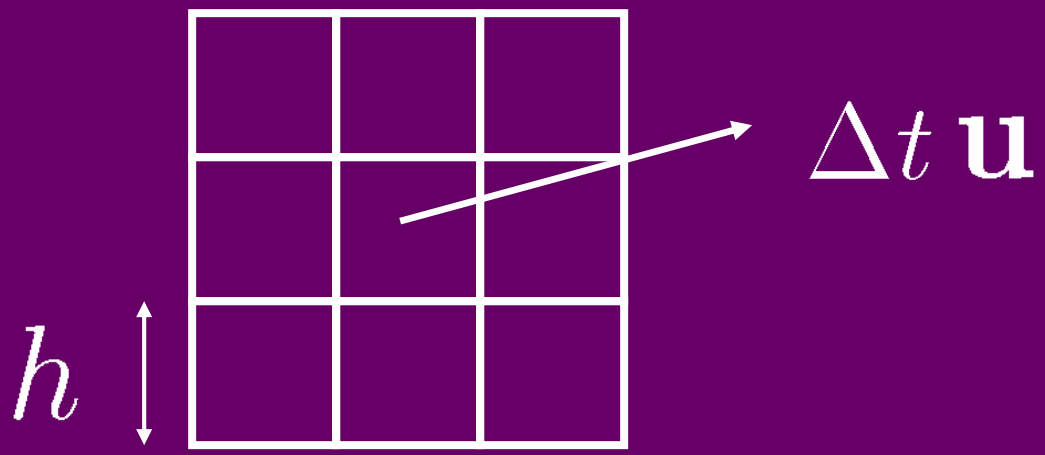
Moving Densities



Velocity known

Moving Densities

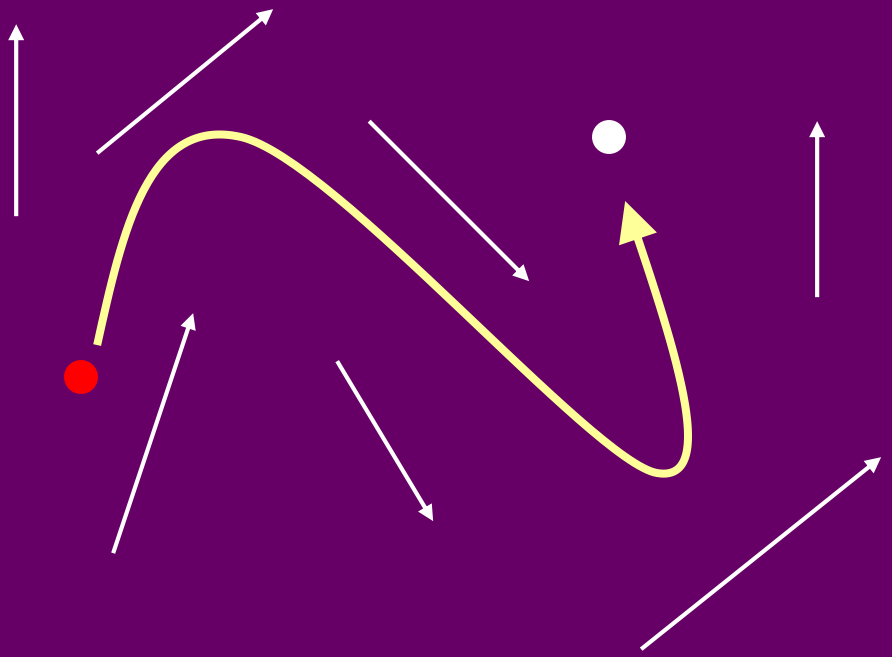
Finite Differences: transfer only between neighbors



Unstable when $\Delta t\ |\mathbf{u}| > h$

Moving Densities

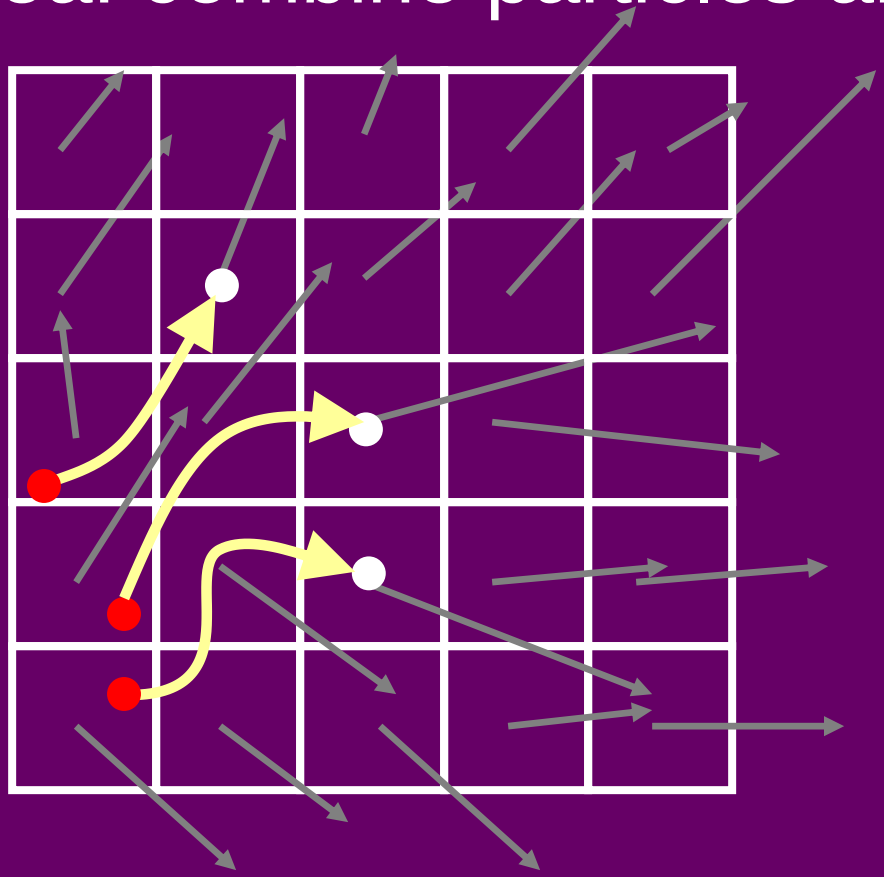
Easy if density defined on particles



Any time step ok

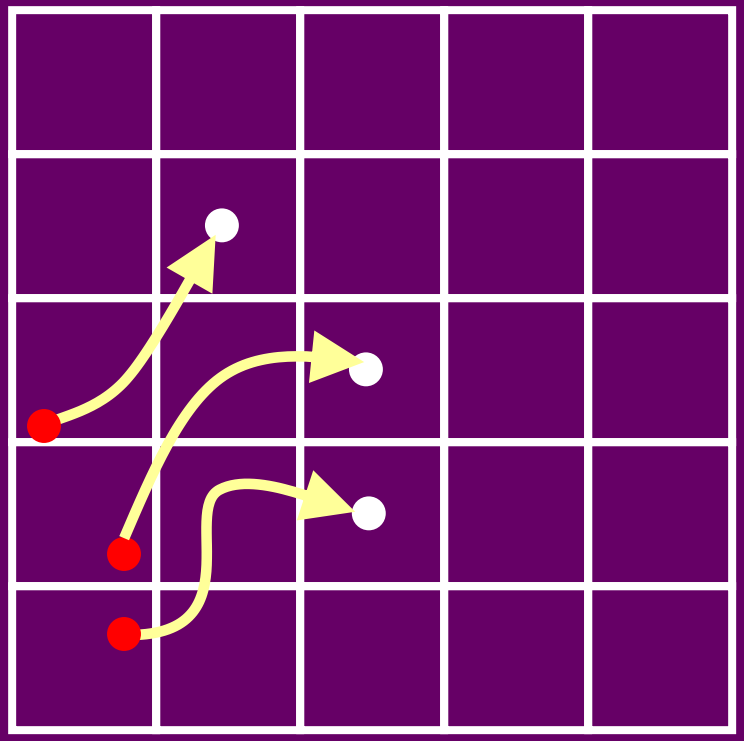
Moving Densities

Key Idea: combine particles and grids



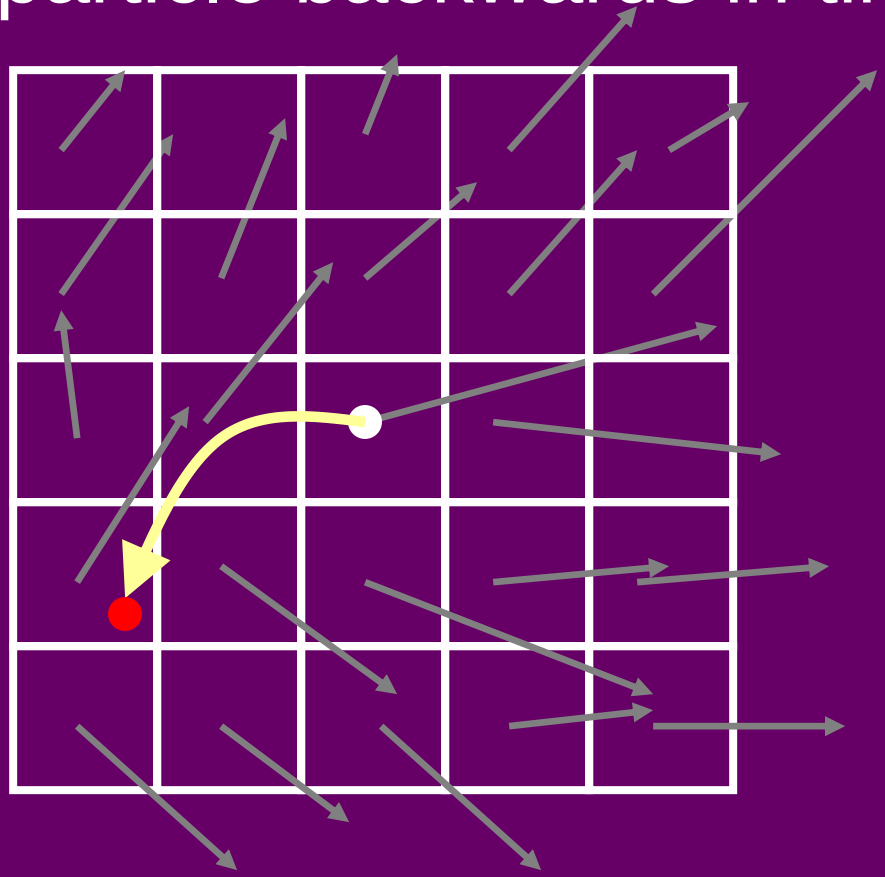
Moving Densities

Key Idea: combine particles and grids



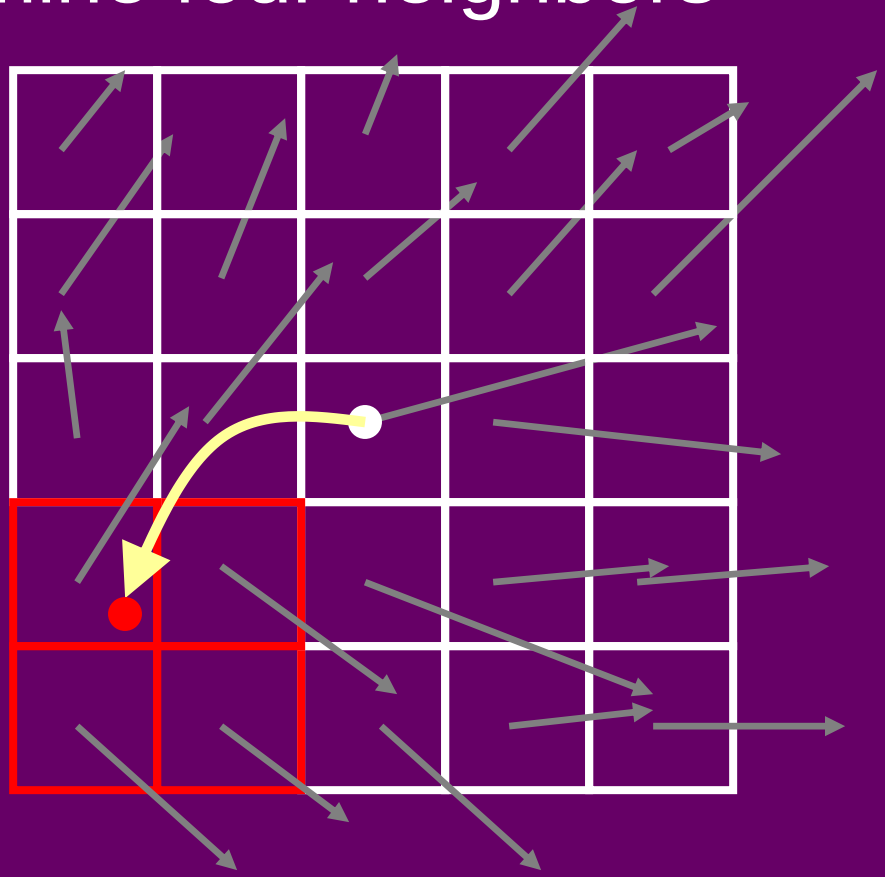
Moving Densities

Trace particle backwards in time



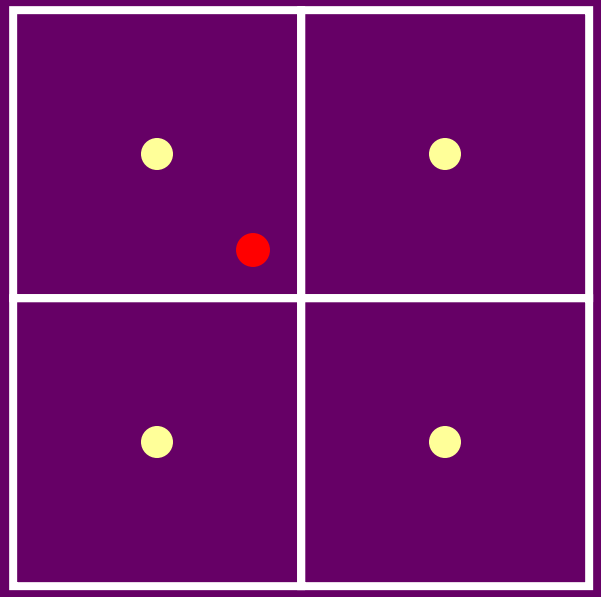
Moving Densities

Determine four neighbors



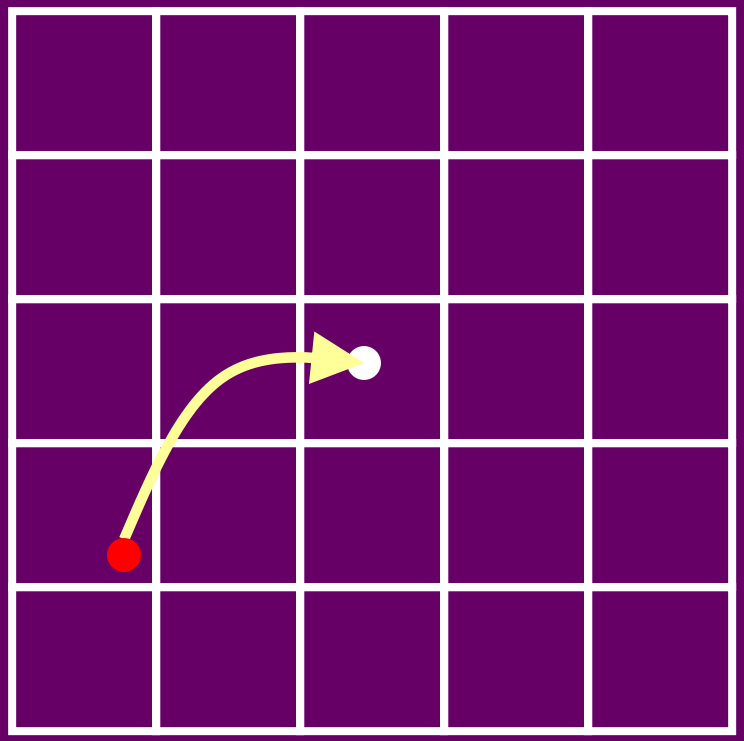
Fluid Mechanics

Interpolate the density at new location



Moving Densities

Set interpolated density at grid location



Requires two grids

Moving Densities

This scheme is unconditionally stable:

$$\rho_{int} = (1 - s)\rho_0 + s\ \rho_1$$

$$\rho_0,\ \ \rho_1 \leq \rho_{max}$$

$$\rho_{int} \leq (1 - s + s)\rho_{max} \leq \rho_{max}$$

$\longrightarrow$ density is always bounded

Computing Velocities

Use same algorithms as for density

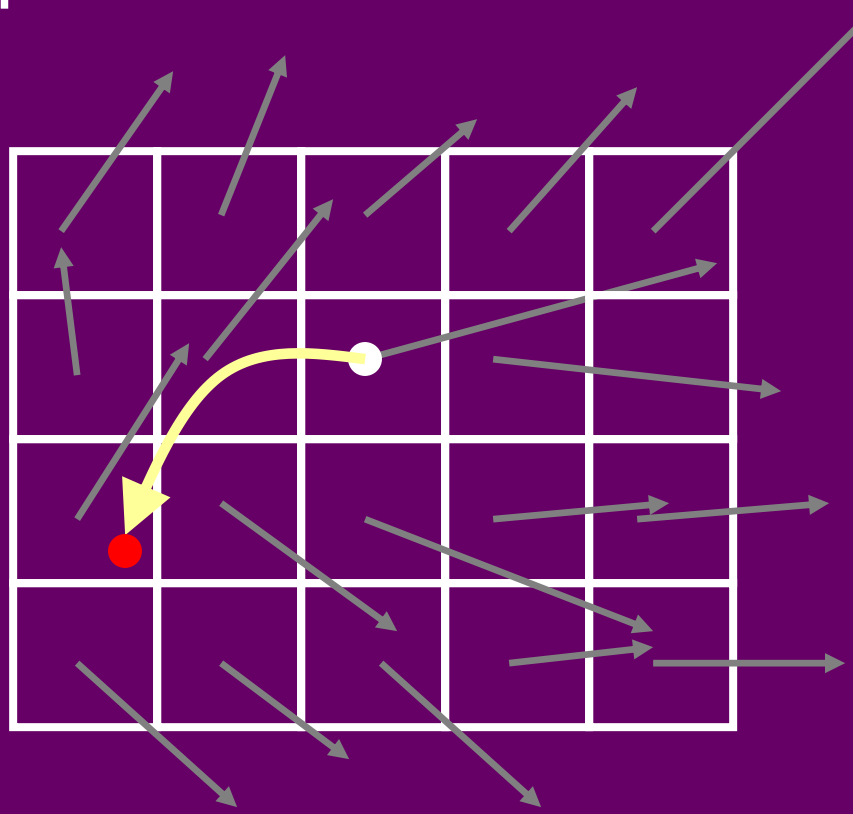
$$\frac{\partial \rho}{\partial t} = -(\mathbf{u} \cdot \nabla)\rho + \kappa \nabla^2 \rho + S$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Velocity is moved by itself

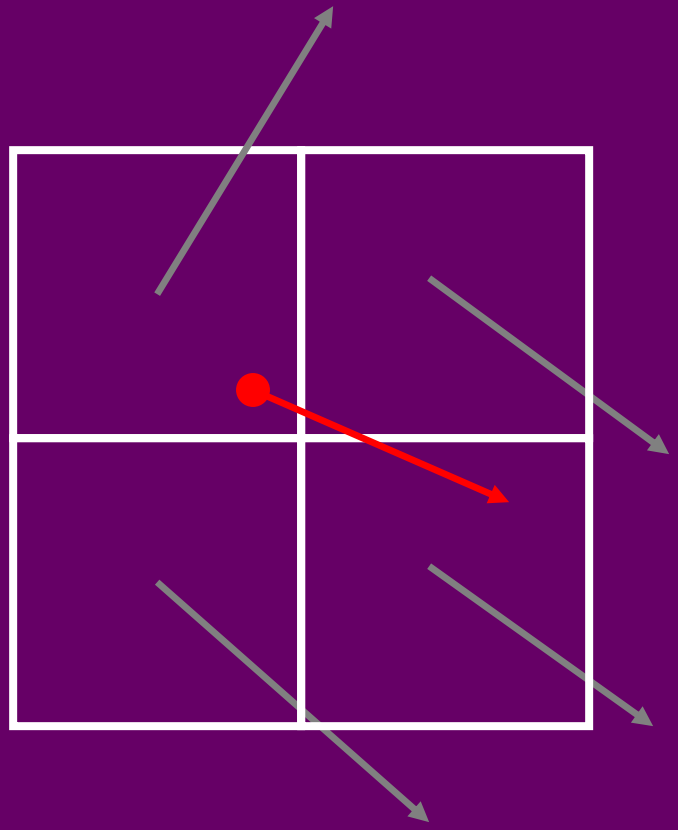
Moving Velocity

Trace particle backwards in time



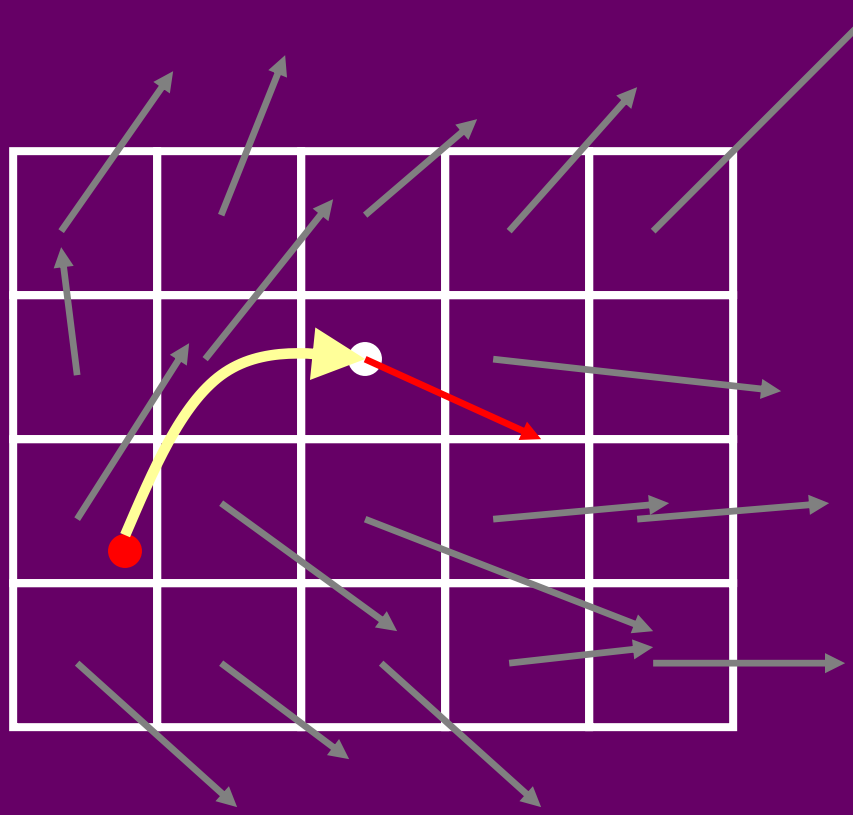
Moving Velocity

Interpolate the velocity at new location



Moving Velocity

Set interpolated velocity at grid location



Requires two grids

Summary

```
UpdateVelocity(U1,U0,F,visc,dt)
    AddForce(U1,U0,F,dt)
    Diffuse(U0,U1,visc,dt)
    Move(U1,U0,U0,dt)
    ConserveMass(U1,dt)
```

Very easy to code. Only need:

- Particle tracer + grid interpolator
- PDE solver (FISHPAK)

Results

Demo time….

Hardware:

sgi Octane Workstation
R12K 300Mhz CPU
3D texture hardware

Future Work

- Handle free boundaries (water)
- Parallel implementation
- Other texture maps
- Handle complex boundaries
- Can use same algorithm for boundary fitted finite element meshes